Monetary Policy Facing Fiscal Indiscipline Under Generalized Timing of Actions[1]

Jan Libich[2]
La Trobe University and CAMA
Petr Stehlík[3]
University of West Bohemia

Abstract

The paper analyzes the interactions between monetary and fiscal policies, both in a single country and a monetary union setting. Focusing on the case of excessively 'ambitious' governments who run structural deficits, we examine under what circumstances ongoing fiscal excesses may spill over and threaten monetary policy outcomes. For that purpose we develop a game theoretic framework that allows for an arbitrary, possibly *stochastic timing* of policy actions. In the framework policy moves can occur with some ex-ante probability distribution, not necessarily with certainty every period as implicitly assumed in most existing settings. Such generalized timing enables us to model various degrees of long-run *monetary commitment* as well as *fiscal rigidity,* the latter potentially heterogeneous across the union member countries. We examine a number of specifications in discrete and continuous time including the widely-used Calvo probabilistic timing. Our main policy contribution lies in deriving the *necessary and sufficient degree* of (long-run) monetary commitment that eliminates socially inferior equilibria. Interestingly, sufficiently strong monetary commitment does not only ensure high credibility of the central bank, but it also indirectly *disciplines* the fiscal policymaker(s) by reducing their payoff from excessive fiscal policies. This is through a credible threat of the central bank engaging in a costly tug-of-war with the government. In contrast, if monetary commitment is insufficiently strong, or there exists a free riding problem in the monetary union, then undesirable outcomes are likely to spill over from fiscal to monetary policy - similarly to the intuition of Sargent and Wallace's (1981) unpleasant monetary arithmetic or Leeper's (1991) active fiscal policy. We conclude by calibrating the game theoretic representation with European Monetary Union data to provide some quantitative predictions regarding the required strength of the European Central Bank's long-run commitment to an explicit inflation target.

Keywords: fiscal-monetary policy interaction; commitment; monetary union; rigidity; Calvo timing; inflation targeting; asynchronous moves; stochastic timing; Battle of the sexes; Game of chicken; **JEL classification:** E61, C70, E42, C72

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[2]Corresponding author: La Trobe University, School of Business, Melbourne, Victoria, 3086, Australia. Phone: (+61) 3 94792754, Fax: (+61) 3 94791654, Email: j.libich@latrobe.edu.au.

[3]University of West Bohemia, Univerzitní 22, Plzeň, 30614, Czech Republic. Phone: (+420) 777 846 682, Fax: (+420) 377 632 602, Email: pstehlik@kma.zcu.cz.

1. Introduction

Fiscal and monetary policies are strongly inter-related, as the actions of one policy affect the outcomes of the other policy. This is true even if the central bank is formally and legally independent. Such inter-dependence implies the following question that has concerned central bankers in many countries (including the European Union and United States), and that will be the main focus of the paper: *Under what circumstances does persistently excessive spending of fiscal policy compromise the anti-inflation credibility of monetary policy, and threaten price stability?*

To examine the interactions of monetary and fiscal policies - in both a single country and a monetary union setting - we propose a game theoretic framework that *generalizes the timing* of the policy actions. The existing literature has, explicitly or implicitly, studied the policy interaction as a standard repeated game.[4] In such a setting all policy moves are: (i) deterministic, ie they occur with certainty at a pre-specified time, (ii) repeated every period, and (iii) simultaneous, ie unobservable by the opponent in real time.

Our framework relaxes these three assumptions that can be viewed as unrealistic in the macroeconomic policy context. It allows for the timing of the policies' moves to be *stochastic*, ie only occur with some probability, and only in some periods. We believe this captures an important aspect of the real world, in which policymakers may often not be able to act as they wish due to various institutional, structural, and political constraints.

In order to separate the effect of stochastic timing of policy actions from the effect of a stochastic macroeconomic environment, our interest lies in the medium/long-run outcomes of the interaction, not the short-run fluctuations. Arguably, these are the first order welfare effects that Sargent and Wallace (1981), Alesina and Tabellini (1987), Nordhaus (1994) and the subsequent literatures were interested in.[5] Because of that, we will not use a specific macroeconomic model. Instead, we will use a 2x2 game theoretic representation that nests the intuition of a number of micro-founded models in the literature (which is discussed in detail in Section 2.2).

To give a simple example of the policy interaction we find most relevant, think of the Battle of the sexes scenario. This game embodies two realistic features: a coordination problem and a policy conflict. Each policy has two available actions labeled 'discipline' and 'indiscipline'. Both the monetary and fiscal policymaker prefer to coordinate their actions to avoid a tug-of-war between them, which would lead to higher macroeconomic volatility (such as the post-reunification Germany situation). Therefore, in a standard one-shot game there are two pure-strategy Nash equilibria, (discipline, discipline) and (indiscipline, indiscipline). Nevertheless, each policymaker prefers a different Nash: the central bank the former to ensure price stability, whereas the government the latter to

[4]See for example Adam and Billi (2008), Eusepi and Preston (2008), Benhabib and Eusepi (2005), Dixit and Lambertini (2003), Leeper (1991), or Sargent and Wallace (1981).

[5]While some papers, including Leeper (1991) and the Fiscal theory of the price level, looked at the stabilization of shocks, their focus was also on *permanent* changes in the policy reactions and behaviour due to the policy interactions. Our long-run focus further implies that by excessive fiscal spending we do *not* mean the governments' responses to the developments in the financial markets in 2007-9, but rather the behaviour that occured prior to the crisis - the persual of structural budget deficits.

buy votes through increased spending. Therefore, in the simultaneous move game the mixed Nash, which leads to inferior outcomes for both policymakers, is a real possibility.[6]

The commonly offered solution is to allow for commitment - Stackelberg leadership - of one policy. Moving first is an advantage in this game as it allows a player to force the opponent to cooperate. It therefore ensures the preferred outcome of the leader to be the unique subgame perfect Nash equilibrium.

The main shortcoming of this simple solution is that the leader 'wins' and the followers 'loses' the Battle of the sexes or the Game of chicken under *all* circumstances, ie regardless of the exact payoffs and discount factors. Therefore, such solution cannot be used to make policy predictions about the outcomes of the monetary-fiscal interactions following a change in some structural, institutional, or preference parameter. This is where our main (non-methodological) contribution lies: our framework refines the conventional insights on the effect of commitment, and enables us to show how macroeconomic outcomes depend on various details of the underlying setup and the timing of the players' actions. The framework even offers a simple way to endogenize the timing.

To motivate the generalized timing, consider the situation in some country with an explicit inflation target. The central bank is committed (on average over the medium-run) to achieve the target, which can only be altered infrequently as it is legislated.[7] In contrast, the government has the opportunity to alter its medium/long-term fiscal stance every year when proposing the budget. In addition, there also exists some positive probability that the fiscal stance can also be changed within the fiscal year: either through election of a new government, macroeconomic developments such as shocks and crises, or because of shifts in the public opinion.

Our framework can capture such timing, and in fact allow for an arbitrary probability distribution of such possible actions. The paper first derives the outcomes of the policy interactions under a general probability distribution, and then depicts in more detail uniform, normal, and binomial distributions of policy moves, the latter following the popular timing of Calvo (1983).[8]

The paper shows that the outcomes of the policy interaction crucially depend on the degree of *monetary commitment* (which measures the inability of the central bank to change its long-run inflation target) *relative* to the degree of *fiscal rigidity* (which measures the inability of the government to put fiscal finances on a sustainable path). If relative monetary commitment is sufficiently strong (explicit), ie above a certain necessary and sufficient threshold $\bar{R}$ we derive, then monetary policy credibility and outcomes will not be threatened by excessively ambitious fiscal policymakers. Relating this outcome to the literature, it can be roughly thought of as the situation of dominant monetary

[6] The policy interaction has also been modelled as a Game of chicken, which will be discussed below.

[7] For example, the 1989 Reserve Bank of New Zealand Act states that the inflation target may only be changed in a Policy Target Agreement between the Minister of Finance and the Governor, and this occurs only when one of them changes (ie roughly every three years or so).

[8] Despite the frequent use of the Calvo (1983) timing in macroeconomic models, this is usually limited to price/wage setting behaviour. The policymakers are still assumed (either explicitly or implicitly) to be able to alter their policy instruments every period. This is true under *discretion, timeless perspective commitment* of Woodford (1999), as well as *quasi commitment* of Schaumburg and Tambalotti (2007). The latter two concepts place restrictions on *how* policy actions can be adjusted, but not the fact that they *can* be adjusted every period.

policy in Sargent and Wallace (1981), active fiscal/passive monetary policy in Leeper (1991), or a Ricardian regime in Woodford (1995).

If however the degree of monetary commitment relative to fiscal rigidity is insufficient (below $\bar{R}$), the central bank is likely to miss its price stability objective. This is due to the spillovers from fiscal policy, and occurs even if the central bank is formally independent from the government and targets the natural rate of output. The intuition is comparable to that of a dominant fiscal regime in Sargent and Wallace (1981), accommodating monetary policy in Sims (1988), active monetary/passive fiscal policy in Leeper (1991), or a non-Ricardian regime in Woodford (1995).

Importantly, what produces valuable insights unobtainable under the standard timing is the fact that the threshold degree of relative commitment $\bar{R}$ is a function of the structure of the economy, and of the preferences of the policymakers. Specifically, in addition to (i) the degree of fiscal rigidity and ambition of each member country, it is also *increasing* in (ii) the country's economic size (ie larger countries carry a greater weight), (iii) the magnitude of the 'conflict cost' associated with the central bank fighting an ambitious government (that is a function of the deep parameters of the underlying macroeconomic model), and (iv) the degree of the central banker's impatience (that is a function of various institutional characteristics of the central banking design).

Perhaps most interestingly, we show that a sufficiently strong long-term monetary commitment may be able to *discipline* fiscal policies (unless the government's ambition is very high, or there exists a substantial free riding problem in the monetary union). The reason for such a 'disciplining effect' is two-fold. First, a more strongly committed central bank will counter-act the expansionary effects of excessive fiscal spending more vigorously. Second, such behavior reduces, or fully eliminates, the short-term political benefits of excessive spending to the government, and hence provides stronger incentives for fiscal consolidation. In the Leeper (1991) framework this can be thought of as an active monetary policy forcing, through the incentives created by its institutional design, fiscal policy to be passive.

This 'disciplining' result seems robust as it holds under all timing distributions of our scenario of interest, and has been derived in Libich et al. (2007) in a different setting.[9] That paper includes a case study in relation to this finding written by Dr Don Brash, the Governor of the Reserve Bank of New Zealand during 1988-2002, in which he argues that *'New Zealand provides an interesting case study illustrating the arguments in the article'*. He describes the policy developments in New Zealand shortly after strengthening monetary commitment by adoption of an explicit inflation target. When the government brought down an excessively expansionary budget in the pre-election period of mid-1990, he was forced to tighten monetary conditions in order to offset the budget's effect, and honour the bank's commitment to the newly legislated inflation target. He documents that these events had a *'profound effect on thinking about fiscal policy in both major parties in Parliament.'* Among other, he recalls that:

> *'Some days later, an editorial in the "New Zealand Herald", New Zealand's largest daily newspaper, noted that New Zealand political parties could no longer buy elections because, when they tried to do so, the newly*

[9]Empirical evidence of this finding from an estimated DSGE model is discussed in Section 8.

instrument-independent central bank would be forced to send voters the bill in the form of higher mortgage rates'.

We conclude by discussing a free riding problem likely to occur in a monetary union, which makes it harder or even impossible for monetary policy to discipline fiscal policy. This is because the potential (political) benefits of excessive fiscal policy are enjoyed predominantly by the indisciplined country itself, whereas the common central bank's punishment via higher interest rates is spread across all member countries, including the disciplined ones. Therefore, if a member country largely ignores the negative externality it imposes on other members than even an infinitely strong future punishment may be insufficient to discipline its fiscal policy. These insights can be related to the debt crises currently under way in Greece and others in the European Monetary Union.

The rest of the paper proceeds as follows. Section 2 presents the monetary-fiscal policy interaction as a game, focusing on scenarios in which there exists a coordination problem between the two policies, and/or an outright conflict between them. Section 3 postulates a game theoretic framework that allows for any deterministic and stochastic timing of moves. Section 4 reports a general result on the outcomes of the interaction for an arbitrary probability distribution of timing, as well as their arbitrary combinations. Section 5 then demonstrates the intuition using specific probability distributions and reports several additional insights. Section 6 shows how our theoretical results can be taken to the data. It first provides a real world interpretation of the main concepts - monetary commitment and fiscal rigidity, and then calibrates the most familiar (Calvo) setup to the case of the European Monetary Union. Section 7 examines four extensions of the analysis, and then reports a fully general result that nests these extensions. Section 8 summarizes and concludes.

2. The Fiscal-Monetary Interaction As a Game

There exist a monetary policymaker, $\mathcal{M}$ (male), and N independent fiscal policymakers, denoted by $\mathcal{F}_n$ (females), where $n \in \{1, 2, ..., N\}$.[10] In a single country setting we have $N = 1$, whereas in a monetary union setting we have $N > 1$.

In the latter, the relative weights of the union members (expressing their economic influence) will be denoted by $w_1, w_2, \ldots, w_N$, such that $\sum_{n=1}^{N} w_n = 1$. Then the overall payoff of the $\mathcal{M}$ policymaker is a *weighted average* of the payoffs obtained from interactions with each individual $\mathcal{F}_n$, using the member's weight w_n. The payoff of each independent government is directly determined by its own actions and those of the common central bank. Indirectly, the actions of other governments will be shown to also have an impact since they determine the action of the central bank, and hence the equilibrium outcomes.

2.1. **Game Theoretic Representation.**

In order to make the game theoretic analysis more illustrative we will examine the policy interaction as a 2x2 game, summarized in the following payoff matrix.

[10]To simplify the notation we will use $\mathcal{F}$ and $\mathcal{M}$ to denote the respective policymakers as well as their policies.

		$\mathcal{F}_n$	
		l	h
$\mathcal{M}$	L	a, w	b, x
	H	c, y	d, z

We can interpret the L and l levels as medium/long-run *discipline*, and the H and h levels as medium/long-run *indiscipline*. In a reduced form model the reader can think of L and H as low and high *average* inflation, and l and h as a *structurally* balanced budget and deficit respectively.

Any analytically solvable (determinate) macroeconomic model of policy interaction can be truncated into such a 2x2 game theoretic representation.[11] The policymakers' payoffs $\{a, b, c, d, w, x, y, z\}$ are then some functions of the deep parameters of the underlying macroeconomic model, ie there is a *mapping* between the selected model and the game theoretic representation. As our focus in this paper is on the game theoretic insights under generalized timing of policy actions, that are applicable to a range of macroeconomic models, we will not examine a specific macroeconomic model here. Nevertheless, we will later discuss an interpretation of these payoffs.

2.2. **Scenarios of Interest.** Naturally, if both policymakers are benevolent and there exist no market imperfections then the socially optimal (L, l) outcome will be the unique Nash equilibrium (abbreviated as NE) of the above long-run game, and this is regardless of the timing of policy actions. However, if some frictions exist and/or (one of) the policymakers have idiosyncratic objectives, then departures from this outcome are likely to occur. This is true in most macroeconomic models of policy interaction, under a range of circumstances.

Following the literature our interest lies in the case in which the government is *ambitious*, ie it attempts to boost output excessively (beyond the potential level).[12] In contrast, the central bank is benevolent and *responsible* trying to achieve a low inflation target and stabilize output at potential.

Due to the distinct objectives, the policies may face a coordination problem, or perhaps even an outright conflict. In particular, we will examine three scenarios of interest arising from some (not necessarily all) macroeconomic models under some (not necessarily all) parameter values. Each of them features two pure and one mixed strategy NE (Figure 1 presents specific examples of each scenario, with the pure strategy NE indicated in bold).

a) **A conflict:** (L, h) and (H, l) are the NE, and each policymaker prefers a different one, namely $\mathcal{M}$ the former and $\mathcal{F}$ the latter. The game has therefore the form of *the*

[11] The working paper version of this article contains a simple macroeconomic model and performs such truncation in the way suggested by Cho and Matsui (2005). They truncate their macro model by setting the L and l levels to the socially optimal values (that are time-inconsistent and do not constitute the equilibrium of their underlying model), whereas the H and h levels to the actual equilibrium (ie time-consistent) levels, that are however socially inferior.

[12] $\mathcal{F}$ ambition may be coming from a desire to get re-elected (and the existence of lobby groups, myopia, unionization, naïve voters etc), as well as from preexisting structural fiscal settings that require deficit financing such as unaffordable welfare/health/pension schemes, or high debt servicing. The latter implies that $\mathcal{F}$ ambition may be 'inherited' (not always something the government has control over), and hence highly persistent.

		$\mathcal{F}_n$	
		l	h
$\mathcal{M}$	L	$0,0$	$\mathbf{2,1}$
	H	$\mathbf{1,2}$	$0,0$

a) Game of chicken

		$\mathcal{F}_n$	
		l	h
$\mathcal{M}$	L	$\mathbf{2,2}$	$0,0$
	H	$0,0$	$\mathbf{1,1}$

b) Pure Coordination

		$\mathcal{F}_n$	
		l	h
$\mathcal{M}$	L	$\mathbf{2,1}$	$0,0$
	H	$0,0$	$\mathbf{1,2}$

c) The Battle of the sexes

Figure 1. Three policy interaction scenarios of interest.

Game of chicken. Specifically, the payoffs satisfy

$$b > c > d \geq a \quad \text{and} \quad y > x > z \geq w, \tag{1}$$

where $(b-a)$ and $(y-w)$ express the players' *conflict costs*, whereas $(b-c)$ and $(y-x)$ are their *victory gains*. Papers that model the policy interaction in such way include Barnett (2001), Bhattacharya and Haslag (1999), Artis and Winkler (1998), and Alesina and Tabellini (1987).

b) **A coordination problem:** (L, l) and (H, h) are the NE, and both policymakers prefer the former. The game has therefore the form of a *Pure coordination game*. Specifically, the payoffs satisfy

$$a > d > \max\{c, d\} \quad \text{and} \quad w > z > \max\{x, y\}, \tag{2}$$

where $(a-b)$ and $(z-x)$ express the players' *mis-coordination costs*, whereas $(a-d)$ and $(z-w)$ are their *coordination gains*. A number of papers on policy interaction feature some type of coordination problem, for example Eusepi and Preston (2008), Chadha and Nolan (2007), Eggertsson and Woodford (2006), Persson et al. (2006), Benhabib and Eusepi (2005), Gali and Monacelli (2005), Dixit and Lambertini (2003) and (2001), van Aarle, et al. (2002), Nordhaus (1994), Petit (1989), or Alesina and Tabellini (1987).

c) **A conflict combined with a coordination problem:** (L, l) and (H, h) are the NE, and each policymaker prefers a different one, namely $\mathcal{M}$ the former and $\mathcal{F}$ the latter. The game has therefore the form of *the Battle of the sexes*. Specifically, the payoffs satisfy

$$a > d > b \geq c \quad \text{and} \quad z > w > x \geq y, \tag{3}$$

where $(a-b)$ and $(z-x)$ express the players' *conflict costs*, whereas $(a-d)$ and $(z-w)$ are their *victory gains*. A large body of literature points to this type of policy interaction, eg Adam and Billi (2008), Branch, et al. (2008), Resende and Rebei (2008), Hughes Hallett and Libich (2007), Benhabib and Eusepi (2005), Dixit and Lambertini (2003) and (2001), Blake and Weale (1998), Nordhaus (1994), Sims (1994), Woodford (1994), Leeper (1991), Wyplosz (1991), Petit (1989), Alesina and Tabellini (1987), or Sargent and Wallace (1981).

As the references demonstrate, each of the three scenarios may arise from (fundamentally) different types of macroeconomic models. Therefore, each model can potentially provide a slightly different mechanism for the possible departure from the socially optimal (L, l) outcome, and a justification for why even a formally independent central bank may find it optimal to monetize the debt to some extent.

One common explanation is the unpleasant monetary arithmetic of Sargent and Wallace (1981), in which seigniorage revenues are required in order to prevent the government from defaulting on its debt. Similarly, in Leeper (1991) and the subsequent literature on the Fiscal theory of the price level, an active $\mathcal{F}$ policy forces $\mathcal{M}$ policy to be passive, and the price level then reacts to $\mathcal{F}$ shocks rather than being autonomously determined by $\mathcal{M}$ policy.[13]

Alternatively, since the policies are substitutes in affecting output in many of the above models (see also Jones (2009) for some empirical evidence), H may be selected to offset some imperfections in the economy and minimize tax distortions, ie 'spread the load' between the policies (see eg Adam and Billi (2008) or Resende and Rebei (2008)). Finally, in Hughes Hallett, et al. (2009), H may under some circumstances (depending on the relative effectiveness of the policies) partly offset the expansionary effect of the deficit, and hence better stabilize output around potential. If the central banker also cares about output stabilization, he may sacrifice some deviation from its inflation target to achieve a less variable output.

All these interpretations imply a conflict as well as a coordination problem, and thus favour the 'Battle scenario' over the other two.[14] Furthermore, Libich et al. (2007) show that the 'Chicken scenario' is unlikely to obtain under a responsible central banker, since he has no structural temptation to inflate if the government is disciplined in the long-term. Because of that, in our discussion of the intuition we will focus on the Battle scenario. Nevertheless, we will also report the results for the remaining scenarios.

2.3. **Outcomes Under Standard Commitment.** Due to the existence of multiple NE in all three scenarios, there exist equilibrium selection problems. While in the 'Coordination scenario' the focal point argument can be used to select the socially optimal NE (L, l), in the remaining two scenarios it is not the case. Since each policymaker prefers a different pure NE standard game theoretic techniques (including evolutionary ones) cannot select between them.

To get sharper predictions the policy interaction has often been studied allowing for commitment - the *Stackelberg leadership* of one player. The following statement is true in all three scenarios considered above, and will provide a benchmark for comparison: Under the standard *static* game theoretic notion of commitment (Stackelberg leadership), the game has a unique outcome that is preferred by the committed player (leader). This is regardless of his discount factor and the exact payoffs (within the constraints (1)-(3) defining each scenario).

Specifically, in the Battle scenario if $\mathcal{M}$ is the Stackelberg leader and $\mathcal{F}$ the follower (often called $\mathcal{M}$ dominance), $\mathcal{M}$'s preferred outcome (L, l) results. This happens for all

[13] The (L, l) outcome can then be interpreted as obtaining under active $\mathcal{M}$ and passive $\mathcal{F}$ policy, the (H, h) outcome under passive $\mathcal{M}$ and active $\mathcal{F}$ policy, and the mixed NE under the policies changing in the active/passive roles.

[14] Probably the closest model to the above game theoretic representation is by Nordhaus (1994). Similarly to our setup, in his macroeconomic model (i) $\mathcal{M}$ is responsible and $\mathcal{F}$ is ambitious, (ii) the focus is on a deterministic steady-state, (iii) a one-shot game in analyzed as a starting point, and (iv) three possible equilibria arise, one preferred by $\mathcal{M}$, one by $\mathcal{F}$, and one inferior for both players (these are comparable to our pure and mixed NE respectively).

parameter values satisfying (3), and even if the central banker is impatient and heavily discounts the future.

In the rest of the paper we examine the outcomes of the interaction allowing for a more general timing of moves, and hence a more general - *dynamic* - concept of commitment. It will become apparent that the conventional conclusions are refined and partly qualified, even if the assumption of a simultaneous initial move is preserved. In particular, whether the (L, l) outcome obtains will depend not only on the relative degrees of commitment, but also on the exact payoffs and the discount rate of the committed player.

3. The Game Theoretic Setup with Generalized Timing of Moves

The framework extends the existing game theoretic literature on *asynchronous move games*, that has primarily examined the simple (deterministic) case of alternating moves.[15] For comparability with the results of the standard repeated game, all our assumptions follow this conventional approach.

Assumption 1. *(i) The timing of all players' moves is exogenous and common knowledge. (ii) All past periods' moves can be observed (ie perfect monitoring). (iii) All players are rational, have common knowledge of rationality, and have complete information about the structure of the game and the opponents' payoffs. (iv) All players move, with certainty, simultaneously every $r \in \mathbb{N}$ periods - starting in (continuous) time $t = 0$.*

Note that all these assumptions can be relaxed. For example, in Section 7.4 we discuss how the timing of moves can be endogenized, ie optimally selected by the players. Let us introduce some terminology regarding the timing of moves and the classification of players.

Definition 1. *Moves made in between the simultaneous moves will be referred to as **revisions**. A player that can make a revision with: (i) some positive probability will be called the **reviser,** and (ii) with zero probability will be called the **committed player** or the **rigid player** - with r expressing his degree of **commitment** or **rigidity**.*[16]

While a game theorist will think in terms of commitment (since his interest lies in the *effect* on the outcomes of the game), a macroeconomist may find it natural to interpret r as either commitment or rigidity (based on the *source* of the inability to move). We will therefore talk about $\mathcal{M}$ commitment, but $\mathcal{F}$ rigidity.

[15]See Cho and Matsui (2005), Wen (2002), Lagunoff and Matsui (1997), or Maskin and Tirole (1988). These papers provide a strong justification and motivation for our general approach; for example, Cho and Matsui (2005) argue that: *'[a]lthough the alternating move games capture the essence of asynchronous decision making, we need to investigate a more general form of such processes'*. Let us stress that our framework with stochastic timing of moves is very different from the so-called stochastic games, in which the random element is some 'state' (see eg Neyman and Sorin (2003) or Shapley (1953)). Recently, Kamada and Kandori (2009) also allow for stochastic revision of actions (the first draft of their paper is dated November 25, 2008, and we became aware of it in August 2009). Their 'revision game' is however static in the sense that the payoffs do not accrue over time, unlike in most macroeconomic settings. Therefore, the intuition, analysis, and applications differ substantially from our dynamic revision game.

[16]Nevertheless, the authors use a very different setting and type of analysis.It is however important to note that due to our focus on the long-run outcomes the 'moves' of $\mathcal{M}$ policy should *not* be interpreted as choosing the interest rate, but instead as deciding on a certain *long-run* stance - eg an average level of inflation. A detailed discussion of this follows in Section 6.1.

Throughout the paper we assume at least one of the players to be the committed player.[17] This is to provide a benchmark for the other player's moves, and examine the timing differences in relative terms. As our focus is on a monetary union with a common central bank but multiple independent $\mathcal{F}$ policymakers, $\mathcal{M}$ will usually have the role of the committed player. Nevertheless, in a single country setting we will also report results for the opposite situation of $\mathcal{F}$ being the rigid player.

The above specification implies that the game consists of a sequence of dynamic games, each r periods long and potentially different. In order to better develop the intuition of our framework we will first examine the r-period game in which the committed player only moves once, and abstract from further repetition. In Section 7.3 we extend the framework into a (finitely or infinitely) repeated setting and show that all of our findings carry over. In fact, it will be evident that we can think of the results from the r-period dynamic game as the worst case scenario, in which repetition does not help the players to coordinate.

Let us now focus on the key aspect of our framework - the timing of the revisions. In particular, one of these moves is of special interest.

Definition 2. *The reviser(s)' **first** revision following each simultaneous move will be labeled **Revision** (with a capital letter). All other revisions will be called **further-revisions**.*

The Revision will have a particular role since it provides the revisers with the first opportunity to react to the committed player's move - observing it. Therefore, the revisers first get a chance to alter their previous action made under imperfect information and potentially punish or reward the committed player.

It is evident that the timing of further-revisions is orthogonal in determining the equilibrium outcomes of the r-period dynamic game, unlike that of the Revisions. In any further-revision the revisers would, regardless of their discount factor, leave their preceding Revision unchanged, since it was made under identical circumstances. Therefore, in the rest of this section we focus on the timing of Revisions.

The probability distribution of an arbitrary (deterministic and stochastic) timing of a Revision can be fully described by a *probability density function,* PDF for short.[18] In terms of the determination of the committed player's payoff and hence equilibrium selection, the following concept will play a crucial role.

Definition 3. *The **individual Revision function***

$$F_n(t) : [0, r] \to [0, 1], \text{ where } F_n(0) = 0, \tag{4}$$

[17] In Libich et al. (2007) this is not imposed, but due to tractability only a simple deterministic case is examined. Also note that the committed player's deterministic moves every r period can be thought of as the *expected* frequency of , ie $r = \frac{1}{1-\theta}$, where θ is the probability he cannot reconsider its long-term stance in any one period.

[18] For a discrete random variable a *probability mass function* is also used, but in order to shorten the exposition, we will describe even discrete distributions by PDFs. Note that (i) this can be done using Dirac delta functions; and (ii) in the rest of the paper we work with cumulative distribution functions so this choice doesn't play any role.

is an arbitrary non-decreasing function summarizing the timing of the n'th reviser's Revision. It is the ***cumulative distribution function (CDF)*** *of the underlying probability distribution, ie it expresses the probability that the reviser has had the opportunity to revise no later than time t. In the $N > 1$ case, the* ***overall Revision function*** *$F(t)$ is the* ***weighted sum of individual CDFs****, denoted* ***wCDF****, with w_n being the weights.*[19]

Several specific examples of $F_n(t)$ are examined below and graphically depicted in Figures 2-5 and 8. These figures also present some related concepts introduced in this section.

Definition 4. *The integral $\int_0^r F(t)\mathrm{d}t$ describes the overall* ***reaction speed*** *of the revisers. The weighted* ***complementary CDF***

$$\int_0^r (1 - F(t))\,\mathrm{d}t = r - \int_0^r F(t)\mathrm{d}t, \tag{5}$$

expresses the ***overall degree of commitment*** *or* ***rigidity*** *of the revisers. Therefore,*

$$\frac{r}{\int_0^r (1 - F(t))\,\mathrm{d}t} \in [1, \infty) \tag{6}$$

is the degree of the committed/rigid player's ***relative commitment*** *or* ***relative rigidity****.*

These concepts are shown graphically in Figure 2. Note that unlike in a standard simultaneous move game, in which only one player can be committed as the Stackelberg leader, in our setup the revisers are also committed (unless $\int_0^r F(t)\mathrm{d}t = 1$). Nevertheless, the degree of their commitment is less than that of the committed player since they can, at least in expectation, move more frequently (unless $\int_0^r F(t)\mathrm{d}t = 0$ which is the case of the standard repeated game).

The way we will go about solving the game is determined by the specific results we are interested in. It is *not* our goal to fully describe all the equilibria of the game under all circumstances. Instead, our interest lies in circumstances under which unique equilibrium selection occurs in our three scenarios with originally multiple equilibria. Specifically, throughout the paper we will be deriving the *necessary and sufficient conditions* under which the dynamic r-period game has a *unique subgame perfect Nash equilibrium* (SPNE) - one that is *Pareto-efficient.*[20] In doing so we will use the following terminology.

Definition 5. *The committed player will be called to* ***win the game*** *if the dynamic r-period game has a unique SPNE, and that SPNE has the committed player's* ***preferred*** *(highest payoff) outcome uniquely on its equilibrium path. Specifically, $\mathcal{M}$'s winning in the Battle scenario will also be referred to as $\mathcal{M}$ policy* ***disciplining*** *$\mathcal{F}$ policy.*

[19]The wCDF is usually called the *probability mixture* in statistics. Let us also note that while we define $F(t)$ on a closed interval $[0, r]$ for ease of exposition, the function relates to Revisions only - the simultaneous moves are not included.

[20]Subgame perfection is a conventional equilibrium refinement that eliminates non-credible threats. A SPNE is a strategy vector (one strategy for each player) that forms a Nash equilibrium after any history.

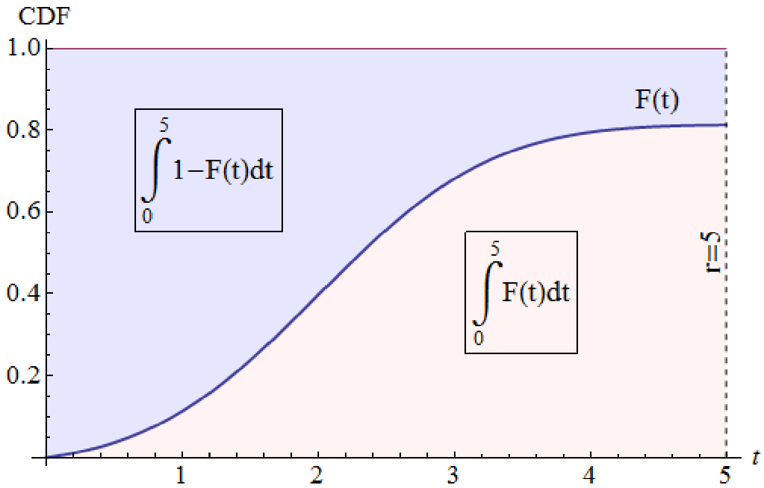

Figure 2. An example of $F(t)$, ie wCDF, under $r = 5$, with the (overall) reaction speed of the reviser(s) indicated.

Note that in all three scenarios if a player wins the game then the other (efficient and inefficient) outcomes of the static game are eliminated from the set of SPNE.

4. Results: Arbitrary Timing of Moves

To make the analysis more illustrative let us streamline it in two ways. First, we will in Sections 4-7.1 abstract from the committed player's discounting the future, and only incorporate it in Section 7.2 showing that the qualitative nature of the results is unchanged. Second, since $\mathcal{M}$ is assumed to be benevolent and his preferred outcome (L, l) is the socially optimal one in the Battle scenario, we will throughout focus on the circumstances under which $\mathcal{M}$ wins the game and disciplines $\mathcal{F}$ policy.

This section will first report a general result that holds for any timing of Revisions. Section 5 will then demonstrate the intuition by examining several specific scenarios, and offer additional insights. This will be complemented by Section 6 which first discusses the real world interpretation of our main concepts, and then reports a calibrated example: the case of the European Monetary Union.

Proposition 1. *Consider the r-period dynamic game without discounting described by either (1), (2), or (3), and by an arbitrary timing of the reviser(s) moves summarized by $F(t)$. The committed/rigid player wins the game if and only if his relative commitment/rigidity is sufficiently high,*

$$\frac{r}{\int_0^r (1 - F(t))\,\mathrm{d}t} > \bar{R} > 1. \tag{7}$$

The relative commitment threshold $\bar{R}$ in the Battle scenario is, under $\mathcal{M}$ and $\mathcal{F}$ being the committed/rigid players respectively,

$$\bar{R} = \frac{a - b}{a - d} \quad \text{and} \quad \bar{R} = \frac{z - x}{z - w}, \tag{8}$$

ie increasing in his conflict cost relative to the victory gain, but independent of the revisers' payoffs.[21]

Proof. As the proof demonstrates the intuition of our framework we reported it here rather than in the Appendix. The committed player only makes one move in the dynamic r-period game. To prove the result it therefore suffices to show that the committed player finds it uniquely optimal to play the action of his preferred NE regardless of the revisers' simultaneous move at $t = 0$. For example if $\mathcal{M}$ is the committed player it suffices to show that L is the unique best response to both l and h simultaneously played by the $\mathcal{F}$ policymaker(s). This is because then $\mathcal{F}$(s) will, in all three scenarios, play their unique best response to L in their every node on the equilibrium path, including the initial move. This is what Definition 5 calls $\mathcal{M}$ policy winning the game.

Focus on the Battle scenario with $\mathcal{M}$ being the committed player. Using backwards induction, it was discussed that any further-revisions would be equivalent to the Revision as they are made under identical circumstances. Moving backwards and considering the Revision, we know that when a particular $\mathcal{F}$ policymaker first gets a chance to respond to $\mathcal{M}$'s move, she will play the very same level played by $\mathcal{M}$. This is because (i) $w > x$ and $z > y$ from (3), and because (ii) $\mathcal{F}$ knows that $\mathcal{M}$ will not be able to alter his action until the end of the r-period dynamic game. In other words, $\mathcal{F}$ will play the static best response to the currently occurring move of $\mathcal{M}$.

Moving backwards, $\mathcal{M}$ takes these anticipated $\mathcal{F}$ Revisions, as well as the expected $\mathcal{F}$ action at $t = 0$, into account in choosing his own initial action. The fact that L is the unique best response to l played in the initial simultaneous move is obvious since $a > c$. Intuitively, there is no policy conflict as the government plays discipline from the outset. However, for L to also be the unique best response to h, ie for the central bank to find it optimal to enter into conflict with an indisciplined government, the following *necessary and sufficient* condition needs to be satisfied

$$\underbrace{b\int_0^r (1 - F(t))\,\mathrm{d}t}_{(L,h)} + \underbrace{a\int_0^r F(t)\mathrm{d}t}_{(L,l)} > \underbrace{dr}_{(H,h)}. \tag{9}$$

The left-hand side (LHS) and the right-hand side (RHS) of this condition report $\mathcal{M}$'s payoffs, under h, from playing L and H respectively. Specifically, the RHS of (9) states that from playing H, $\mathcal{M}$ will get the payoff d throughout the game (in which case there is no policy conflict as $\mathcal{M}$ concedes without a fight).

In contrast, the LHS of (9) states that if $\mathcal{M}$ plays L he will get the conflict payoff b for interactions with $\mathcal{F}$s that have not been able to revise yet, and the victory payoff a with those who have (and have therefore switched to their best response l). The two elements on the LHS can be thought of as $\mathcal{M}$'s initial investment to win, which is costly, and a subsequent reward for winning the game. Specifically, b expresses the magnitude

[21] The remaining thresholds are the following: in the Chicken scenario $\bar{R} = \frac{b-a}{b-c}$ and $\bar{R} = \frac{y-w}{y-x}$ (ie also the conflict cost relative to the victory gain), and in the Coordination scenario $\bar{R} = \frac{a-b}{a-d}$ and $\bar{R} = \frac{w-y}{w-z}$ (ie mis-coordination cost relative to the coordination gain). Obviously, if the payoffs are symmetric then the two $\bar{R}$ values within each scenario are equivalent. For instance, using the specific payoffs in Figure 1 all six thresholds $\bar{R}$ equal 2.

of the cost and $\int_0^r (1 - F(t))\, \mathrm{d}t$ expresses the duration of the cost as given by the area *above* $F(t)$. Similarly, the payoff a expresses the magnitude of the reward and $\int_0^r F(t)\mathrm{d}t$ expresses the duration of the reward as given by the area *below* $F(t)$. Both of these are relative to what $\mathcal{M}$ would have received by avoiding the conflict and accommodating excessive $\mathcal{F}$ policy from the start, dr on the RHS.

Using (5) and rearranging (9) we obtain $\frac{r}{\int_0^r (1-F(t))\mathrm{d}t} > \frac{a-b}{a-d}$ as claimed in the Proposition. Realizing that the necessary and sufficient conditions for the other scenarios, as well as for the case of $\mathcal{F}$ being the rigid player are derived analogously, and hence only differ in the value of the threshold $\bar{R}$, completes the proof. □

Unlike in the case of standard static commitment discussed in Section 2.3, the committed player may not always win the game. To do so he needs to be *sufficiently strongly* committed relative to the reviser, where the threshold $\bar{R}$ is a function of his payoffs and hence various deep parameters of the macroeconomic model. In the game theoretic representation it is about the cost of the potential conflict or mis-coordination relative to the gain of securing the preferred outcome.

This implies that allowing for dynamics refines the conclusions made under the standard concept of commitment, where the outcomes are not contingent on the exact payoffs. As such, our framework may provide valuable information to the policymakers, as they can consider their 'optimal' degree of commitment. We will in Section 7.4 briefly examine such endogenous determination of commitment and timing.

The proposition further highlights the importance of *relative* commitment/rigidity - what matters is how frequently/likely a player can move relative to the opponent(s). Graphically, it is about the relative size of the areas below and above $F(t)$ as shown in Figure 2. Note that this insight is obtained neither under the standard commitment concept, nor in models on optimal $\mathcal{M}$ commitment that abstract from $\mathcal{F}$ policy (eg Schaumburg and Tambalotti (2007)).

For completeness, let us discuss what happens if the condition in (7) is *not* satisfied, ie if neither player's relative commitment/rigidity is sufficient. Then neither player wins the game according to Definition 5 as the dynamic r-period game has *multiple SPNE*. Specifically in the Battle scenario, there will be (i) the socially optimal SPNE with (L, l) uniquely on the equilibrium path, but also (ii) the socially inferior with (H, h) throughout the equilibrium path. In addition, there are potentially (iii) other SPNE featuring some (pure or mixed) combination of (L, l, H, h) on the equilibrium path, dependent on the exact values of the players' commitment/rigidity and their payoffs.

This implies that if (7) does not hold the socially optimal outcomes *may* or *may not* obtain. Considering the multiplicity region is beyond the scope of the presented paper, but intuitively in an evolutionary setting the higher a player's relative commitment/rigidity the 'closer' he gets to his preferred outcome since its basin of attraction is larger.[22]

[22] For more see Basov, Libich and Stehlík (2009) that examine stochastically stable states in a similar framework.

5. Results: Specific Timing Distributions

In order to further develop the intuition and provide additional insights, we will examine several timing specifications summarized in the following table.

Case	Moves	Time
1	deterministic	discrete
2	uniformly distributed	continuous
3	binomially distributed (Calvo)	discrete
Ext 1	combinations (including normally distributed)	continuous

In each case we first examine the monetary union setting with a single $\mathcal{M}$ and any number N of independent $\mathcal{F}$ policymakers. The latter can be heterogenous not only in terms of their degree of $\mathcal{F}$ rigidity, but also in terms of their economic size w_n. The conditions for the special case of a single country setting with $N = 1$ is then also reported (as it is nested in the general solution such sequencing will minimize the number of equations).

In each case the following steps will be made - both mathematically and graphically. First, the underlying probability distributions of the timing of the Revisions are postulated. Second, the individual and overall Revision functions $F_n(t)$ and $F(t)$ are summarized. Third, their integrals are derived. Fourth, these are rearranged and substituted into the general condition (7) to obtain the specific condition for each case.

5.1. Case 1: Deterministic Moves.

This case provides a benchmark, and in line with Tobin (1982) it allows for the *frequency of moves* to differ across players.[23] Specifically, each $\mathcal{F}$ policymaker n moves with a constant frequency - every $t = jr_n^{\mathcal{F}}$ periods, where $j \in \mathbb{N}, r_n^{\mathcal{F}} \in \mathbb{N}$, and $\lfloor r/r_n^{\mathcal{F}} \rfloor = r/r_n^{\mathcal{F}}, \forall n$.[24] The latter assumption that the floor equals the integer value implies $r_n^{\mathcal{F}} \leq r$, as well as synchronization of the simultaneous moves across all policymakers, ie Assumption 1(iv) to hold.

The individual and overall Revision functions, ie the CDFs and the wCDF, have the following specific form (see Figure 3 that graphically depicts these as well as the timeline of the game)

$$F_n(t) = \begin{cases} 0 & \text{if} \quad t < r_n^{\mathcal{F}}, \\ 1 & \text{if} \quad t \geq r_n^{\mathcal{F}}, \end{cases} \qquad \text{and} \qquad F(t) = \sum_{n: r_n^{\mathcal{F}} \leq t}^{N} w_n. \tag{10}$$

Integrating $F(t)$ from (10) over $[0, r]$ we obtain

$$\int_0^r F(t)\mathrm{d}t = w_1(r - r_1^{\mathcal{F}}) + w_2(r - r_2^{\mathcal{F}}) + ... + w_N(r - r_N^{\mathcal{F}}) = r - \sum_{n=1}^{N} w_n r_n^{\mathcal{F}}.$$

Using (5) implies the specific form of the necessary and sufficient condition in (7), namely

$$\frac{r}{\sum_{n=1}^{N} w_n r_n^{\mathcal{F}}} > \bar{R}. \tag{11}$$

[23] In his Nobel lecture Tobin observed that '*Some decisions by economic agents are reconsidered daily or hourly, while others are reviewed at intervals of a year or longer. It would be desirable in principle to allow for differences among variables in frequencies of change...*'.

[24] Note that this case nests the conventional repeated game under $r_n^{\mathcal{F}} = r, \forall n$.

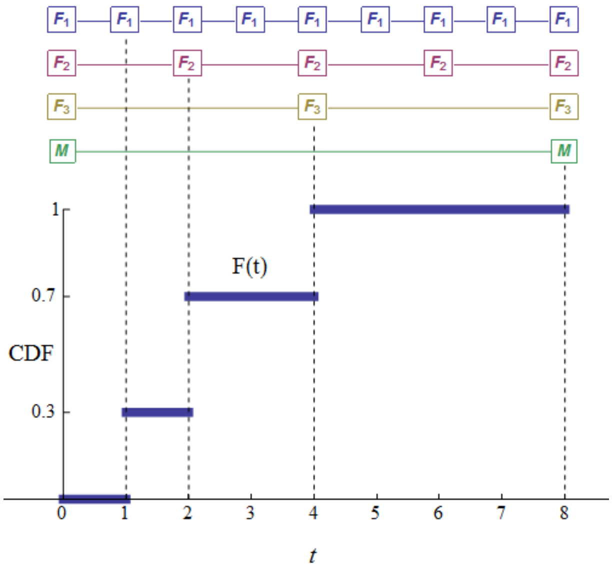

FIGURE 3. The timeline and $F(t)$ of Case 1 featuring $r = 8, \mathcal{M}$ as the committed player, and three $\mathcal{F}$ policymakers with weigths $w_1 = 0.2, w_2 = 0.5, w_3 = 0.3$ and rigidities $r_1^{\mathcal{F}} = 1, r_2^{\mathcal{F}} = 2, r_3^{\mathcal{F}} = 4$.

In a single country setting, $N = 1$, the condition becomes

$$\frac{r}{r^{\mathcal{F}}} > \bar{R} = \frac{a-b}{a-d},$$

where $\frac{r}{r^{\mathcal{F}}}$ expresses $\mathcal{M}$'s relative commitment in Case 1. Analogously, if the roles are reversed and $\mathcal{F}$ is the rigid player, for her to win the necessary and sufficient condition becomes

$$\frac{r}{r^{\mathcal{M}}} > \bar{R} = \frac{z-x}{z-w},$$

where $r^{\mathcal{M}}$ is the analog of $r^{\mathcal{F}}$ defined above. Both $\bar{R}$ thresholds are obviously identical to those reported in Proposition 1 for the Battle scenario.

5.2. **Case 2: Uniformly Distributed Moves.** Consider some g_n and h_n, such that $0 \leq g_n < h_n \leq r, \forall n$, as the minimum and maximum $\mathcal{F}$ rigidity of the n'th country respectively. Further assume that each $\mathcal{F}$ policymaker n has a Revision with a uniformly distributed probability on the interval $[g_n, h_n] \subseteq [0, r]$. The individual and overall Revision functions have the following specific form (see Figure 4 for a plot, where the solid line in the timeline denotes moves with probability 1 in a given period, and the dashed line moves with probability less than 1)

$$F_n(t) = \begin{cases} 0 & \text{if} \quad t \in [0, g_n), \\ \frac{t-g_n}{h_n-g_n} & \text{if} \quad t \in [g_n, h_n), \\ 1 & \text{if} \quad t \in [h_n, r], \end{cases} \qquad \text{and} \qquad F(t) = \sum_{n=1}^{N} w_n F_n(t). \tag{12}$$

Integrating $F(t)$ from (12) over $[0,r]$ we get

$$\int_0^r F(t)\mathrm{d}t = \sum_{n=1}^{N} w_n[r - \frac{1}{2}(g_n + h_n)] = r - \frac{1}{2}\sum_{n=1}^{N} w_n(g_n + h_n), \tag{13}$$

Using (5) implies the specific form of the necessary and sufficient condition in (7), namely

$$\frac{r}{\frac{1}{2}\sum_{n=1}^{N} w_n(g_n + h_n)} > \bar{R}. \tag{14}$$

In a single country setting, $N = 1$, the condition becomes

$$\frac{r}{\frac{1}{2}(g+h)} > \bar{R}.$$

If $\mathcal{F}$ is the rigid player the condition is the same with g and h relating to player $\mathcal{M}$.

5.3. **Case 3: Binomially Distributed Moves.** The Calvo (1983) timing has become increasingly used in the macroeconomic literature when modelling the moves of the price/wage-setters. We believe it is also useful in modeling the timing of policy actions, and will therefore use it for calibration in Section 6.2. Assume that each $\mathcal{F}$ policymaker n moves every uniformly distributed discrete period t (for example $t \in \mathbb{N}$), but only with probability $(1-\theta_n)$. This probability is independent across time and players.[25]

The individual and overall Revision functions have the following specific form (see Figure 5 for a graphical depiction)

$$F_n(t) = \sum_{i=0}^{\lfloor t \rfloor -1} \theta^i (1-\theta) = 1 - \theta^{\lfloor t \rfloor} \quad \text{and} \quad F(t) = \sum_{i=0}^{\lfloor t \rfloor -1}\sum_{n=1}^{N} w_n (1-\theta_n)\theta_n^i = 1 - \sum_{n=1}^{N} w_n \theta_n^{\lfloor t \rfloor}. \tag{15}$$

Integrating $F(t)$ from (15) over $[0,r]$ we obtain

$$\int_0^r F(t)\mathrm{d}t = r - \sum_{i=0}^{r-1}\sum_{n=1}^{N} w_n \theta_n^i = r - \sum_{n=1}^{N} w_n \frac{1-\theta_n^r}{1-\theta_n}. \tag{16}$$

Using (5) implies the specific form of the necessary and sufficient condition in (7), namely

$$\frac{r}{\sum_{n=1}^{N} w_n \frac{1-\theta_n^r}{1-\theta_n}} > \bar{R}. \tag{17}$$

In a single country setting, $N = 1$, the condition becomes

$$\frac{r}{(1+\theta+\theta^2+\ldots+\theta^{r-1})} > \bar{R}. \tag{18}$$

By inspection of (11), (14), and (17), the minimum r value that satisfies these conditions is increasing in the country's weight w_n as well as in the degree of $\mathcal{F}$ rigidity, which is $r_n^{\mathcal{F}}$ in Case 1, $\frac{(g_n+h_n)}{2}$ in Case 2, and θ_n in Case 3. The following proposition summarizes these findings.

[25]Note that if $\theta_n = 1, \forall n$, then we get $F(t) = 0$, which corresponds to Case 1 under $r_n^{\mathcal{F}} = r, \forall n$, and hence the conventional repeated game. If $\theta_n = 0$ we get Case 1 with $r_n^{\mathcal{F}} = 1$.

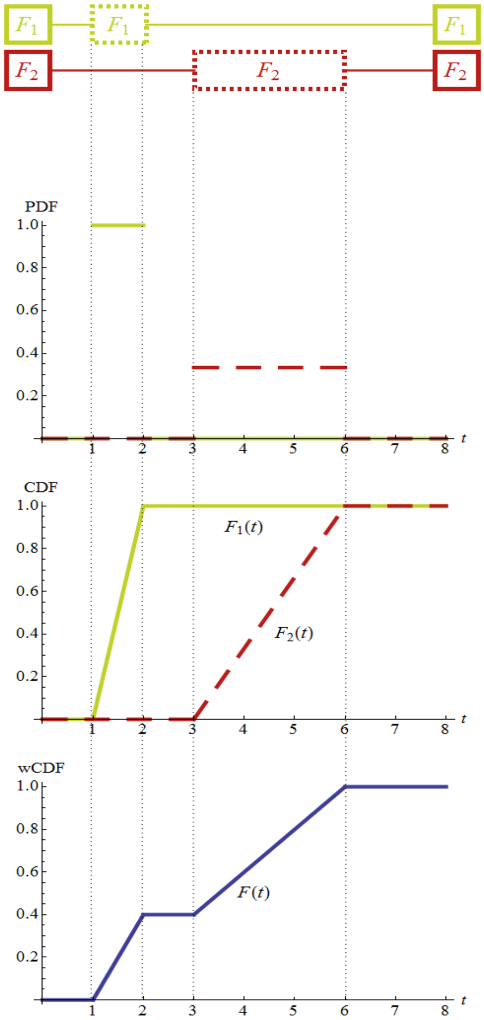

FIGURE 4. The timeline, PDFs, $F_n(t)$ (ie CDFs), and $F(t)$ (ie wCDF) of Case 2 featuring $r = 8$ and two revisers. They have $w_1 = 0.4, w_2 = 0.6, g_1 = 1, h_1 = 2, g_2 = 3, h_2 = 6$, ie uniformly distributed probability of Revisions on intervals $[1, 2]$ and $[3, 6]$ respectively.

Proposition 2. *The greater the* ***economic size*** *of the member country w_n, the more her $\mathcal{F}$* ***rigidity*** *(and hence ambition) increases the necessary and sufficient degree of $\mathcal{M}$ commitment r under which $\mathcal{M}$ wins the game and disciplines the $\mathcal{F}$ policymaker(s).*

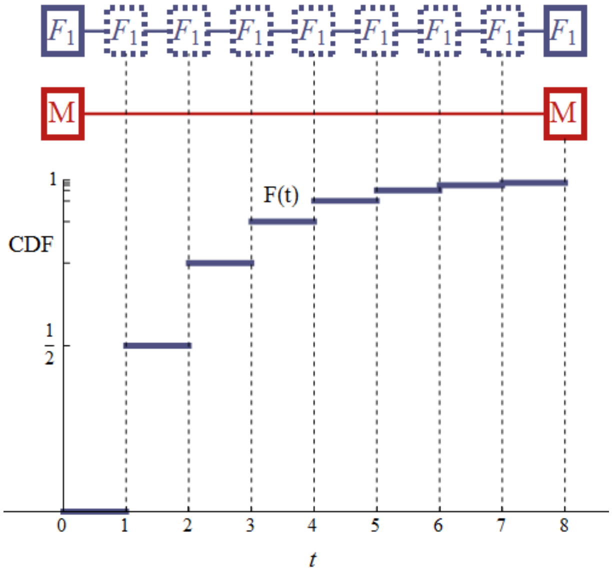

FIGURE 5. The timeline and $F(t)$ of Case 3 featuring $r = 8$, $\mathcal{M}$ as the committed player, and one $\mathcal{F}$ policymaker with $\theta = \frac{1}{2}$.

Proof. Rewriting condition (9) in terms of $F_n(t)$ rather than $F(t)$ one obtains

$$\sum_{n=1}^{N} w_n \left(b \int_0^r (1 - F_n(t))\,\mathrm{d}t + a \int_0^r F_n(t)\mathrm{d}t \right) > dr. \tag{19}$$

Rearranging and using $\bar{R} = \frac{a-b}{a-d}$ yields

$$\frac{r}{\sum_{n=1}^{N} w_n \int_0^r (1 - F_n(t))\,\mathrm{d}t} > \bar{R}.$$

The fact that the denominator is increasing (and hence the whole fraction decreasing) in w_n completes the proof by inspection. □

Intuitively, the greater a union member's economic influence, and the more fiscally rigid the member is, the more she increases the required degree of $\mathcal{M}$ commitment that will discipline her, and other member countries. This is in order to provide sufficient incentives for $\mathcal{F}$ consolidation - sufficiently strong punishment for $\mathcal{F}$ indiscipline. Such punishment will discourage the government(s) from running structural deficits by strongly counter-acting their expansionary effect.

6. Real World Interpretation and Application

We have kept the focus on the game theoretic insights in terms of the policy interaction - allowing for various (deterministic and stochastic) timing scenarios. This section will attempt to bring them to life. It will first provide a real world interpretation of the main variables of our analysis. It will then apply the results to the case of the European Monetary Union (EMU) - by calibrating Case 3 with the EMU data.

6.1. **Interpretation.** Our analysis up to this point has been general enough to be applicable to a wide range of macroeconomic models of $\mathcal{M}$-$\mathcal{F}$ policy interaction. We have only assumed, in line with most of the literature surveyed in Section 2.2, that (i) the $\mathcal{M}$ policymaker is responsible, whereas the $\mathcal{F}$ policymaker is excessively ambitious, and, because of that (ii) there exist a coordination problem and/or a policy conflict. Further, our attention has been on medium/long-run outcomes of such policy interaction in order to separate the effect of stochastic timing from a stochastic macroeconomy (shocks).

Such focus implied that the instrument of $\mathcal{M}$ policy should not be interpreted as a choice of the interest rate, but instead as deciding on a certain *average stance* - average level of inflation. Similarly, the $\mathcal{F}$ policy instrument represents choosing the long-run stance of F policy, which includes (but is not limited to) the average size of the budget deficit and debt.

This points to the interpretation of our main concepts - $\mathcal{M}$ commitment and $\mathcal{F}$ rigidity. They both relate the players' inability to alter their previous long-run stance, and hence the question one needs to answer is the following: What are the real world factors that prevent the policymakers from changing the long-run stance at will?

It can be argued that such inability is due to the fact that some important features affecting the policy decisions are *legislated*. Therefore, $\mathcal{M}$ commitment and $\mathcal{F}$ rigidity can be interpreted as the degree of *explicitness* with which the settings and/or targets of the respective policies are stated in the legislation or central banking statutes. The underlying assumption is that the more explicitly a certain policy setting/goal is grounded and visible to the public, the less frequently it can be altered (in a deterministic sense), or the less likely it is to be altered (in a probabilistic sense).

In terms of $\mathcal{M}$ policy, one example of an explicit $\mathcal{M}$ commitment used by a number of countries is a legislated numerical inflation target. Such commitment means that the central bank cannot reconsider the *long-run* inflation level arbitrarily. For example, the 1989 Reserve Bank of New Zealand Act states that the inflation target may only be changed in a Policy Target Agreement between the Minister of Finance and the Governor The Act also states that the Governor may be fired if inflation were to deviate from the target in the medium-term.

In terms of $\mathcal{F}$ policy, there are a number of factors that make an excessively ambitious stance rigid (persistent). For example, it is various political economy reasons (lobby groups, myopia, unionization, naïve voters) or structural features (aging population, pay-as-you-go health and pension systems, welfare schemes, high outstanding debt etc). All these determine the degree of $\mathcal{F}$ ambition, and the extent to which these are grounded in the legislation (or political culture) then affects the degree of $\mathcal{F}$ rigidity, which is postulated quantitatively in the next section.[26]

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[26]Leeper (2009) makes strong arguments for improvements in the design of $\mathcal{F}$ policy along the lines of those implemented in $\mathcal{M}$ policy over the past two decades. These would have two effects in our framework. First, legislating them would make the long-run stance more rigid (ie increase the value of θ in Case 3). Second, it would to a large extent eliminate $\mathcal{F}$ ambition and the incentive of governments to run structural deficits. Therefore, the latter change would yield a game in which (L, l) is the outcome in both the static and dynamic game. Put differently, our analysis implies that if both policymakers

6.2. **Calibration: the EMU.** $\mathcal{M}$ policy in the EMU is conducted by a common $\mathcal{M}$ authority, the European Central Bank (ECB). In contrast, each country has an independent $\mathcal{F}$ policy.[27] In order to consider multiple $\mathcal{F}$ policymakers the ECB will be the committed player.

As of the writing of this paper, there are fifteen member countries that have adopted the common currency Euro (the so-called Eurozone), namely Austria, Belgium, Cyprus, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Malta, the Netherlands, Portugal, Slovenia, and Spain. Therefore, we set $N = 15$.

Among them, two types of heterogeneity are arguably the most important. First, it is the economic size that differs greatly across the member countries. Second and more crucially, it is the degree of $\mathcal{F}$ rigidity and ambition. As both types of $\mathcal{F}$ heterogeneity are present in Case 3, and the Calvo probabilistic timing is the most widely used type of rigidity in the literature, we will utilize it here (the other cases yield comparable outcomes).

Recall that the necessary and sufficient condition for $\mathcal{M}$ to discipline the $\mathcal{F}$ policymakers in the Calvo setting and the Battle scenario is reported in (17), namely

$$\frac{r}{\sum_{n=1}^{N} w_n \frac{1-\theta_n^r}{1-\theta_n}} > \bar{R} = \frac{a-b}{a-d}, \tag{20}$$

In this case θ_n can be interpreted as the probability that $\mathcal{F}$ is *unable* to consolidate her actions, even if it is her optimal play (ie after observing the central bank's determination to fight regardless of the associated costs). As the previous section discussed, there exist a number of obstacles for a government to consolidate its $\mathcal{F}$ actions and put them on a sustainable path, even if it wishes to do so.

The question of how to best calibrate θ_n in (20) therefore amounts to the following: What is the probability that the government of country n will embark on a $\mathcal{F}$ policy stance that is balanced over the long-term - conditional upon deciding that it is the optimal thing to do? We believe such probability can best be derived from $\mathcal{F}$ outcomes of the (recent) past.

Specifically, we propose the following function for assigning a θ_n value to the EMU members

$$\theta_n = \begin{cases} \frac{\alpha S_n}{\alpha S_n - 1} & \text{if} \quad S_n \leq 0, \\ 0 & \text{if} \quad S_n > 0, \end{cases} \tag{21}$$

where α is some positive constant (that determines the exact slope of θ_n), and S_n is the arithmetic mean of country n's $\mathcal{F}$ surplus as a percentage of the gross domestic product (GDP) over the period 2001-2006 (inclusive) using Eurostat data, see Appendix A. This implies that $S_n > 0, S_n < 0$, and $S_n = 0$ indicate an *average* surplus, deficit and balanced budget respectively. We start the sample in 2001 rather than in 1999 (the year in which the Euro was officially adopted) in order to exclude the idiosyncratic effects of the Maastricht criteria on $\mathcal{F}$ policy outcomes around the time of the Euro's adoption. Similarly, we do not include the 2007-8 data in order to exclude the effects of

are responsible then there is no policy conflict, and hence the relative degrees of $\mathcal{M}$ commitment and $\mathcal{F}$ rigidity do not affect *long-term* macroeconomic outcomes.

[27]While the Maastricht criteria provide some constraints on the independence of the member goverments, these are neither strict nor, as past experience shows, strictly binding.

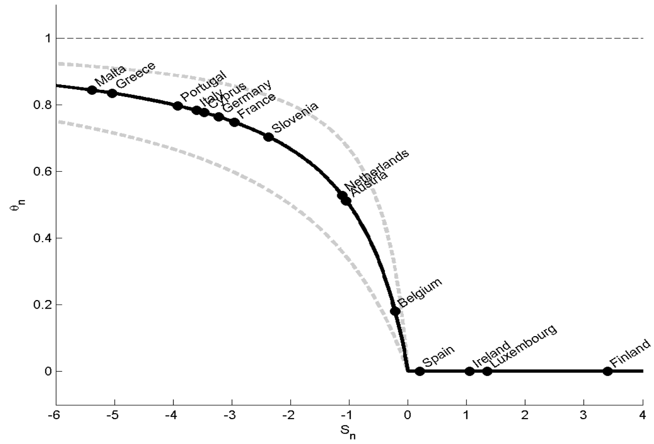

FIGURE 6. Dependence of F rigidity θ_n on the budget surplus to GDP ration S_n, see (21), depicting the EMU countries. The solid line reports the baseline case $\alpha = 1$, the upper and lower lines depict $\alpha = 2$ and $\alpha = \frac{1}{2}$ respectively.

the global financial crisis. Nevertheless, this time frame seems sufficient to suggest the medium/long-run stance of $\mathcal{F}$ policy in these countries.[28]

The choice of the most realistic α depends on the interpretation of the length of each period, t, and the frequency of the central bank's long-run moves, r. It was stressed above that we examine the trend outcomes of the policy interaction. Therefore, we interpret t as one *year*, which is the frequency of the government proposing and implementing the budget, and hence getting a chance to become fiscally sound from that point onwards.

As a baseline we set $\alpha = 1$ in (21) - see Figure 6 for a plot that shows the resulting θ_n values for the EMU countries. Such parametrization implies that a country such as Austria with an average deficit of 1% of GDP, $S_n = -1$, has a 50% probability ($\theta_n = \frac{1}{2}$) of consolidating $\mathcal{F}$ finances each year, whereas a countries with $S_n = -3$ such as Germany or France only have a 25% probability of doing so each year ($\theta_n = \frac{3}{4}$). For obvious reasons, θ_n in (21) is truncated by zero from below for countries with a surplus on average, $S_n > 0$. This means that the four such countries in the sample - Finland, Ireland, Luxemburg, and Spain - are assigned the value of $\theta_n = 0$.

If the reader, like the authors, finds the values implied by $\alpha = 1$ overly optimistic in terms of the $\mathcal{F}$ consolidation opportunities, ie if the driving forces of $\mathcal{F}$ indiscipline are more persistent, s/he may want to select some $\alpha > 1$, which will increase the value of θ_n. Conversely, a lower value of α will imply a more favourable outlook (see Figure 6 that also depicts $\alpha = 2$ and $\alpha = \frac{1}{2}$).

In terms of the weights w_n, we use each country's real GDP share of the EMU's total. Specifically, using Eurostat data, we calculate the average annual GDP for each EMU

[28]Including the size of each country's debt as a percentage of GDP into the specification of S_n would not change the quantitative nature of the results.

member country over 2001-2006, and divide it by the EMU's average over that period (see Appendix A).

As discussed in Section 2.1, the fraction $\frac{a-b}{a-d}$ is some function of the deep parameters of the underlying macroeconomic model. Since each of our three scenarios can be generated via fundamentally different models, it is not possible to provide general mapping between the deep parameters and payoffs. Nevertheless, it can be done for a specific macroeconomic model, as Libich et al. (2007) demonstrate, following the approach of Cho and Matsui (2005).

They use a simple reduced-form model reminiscent of Nordhaus (1994), in which both policymakers have the standard quadratic utility over inflation and output stabilization, but they differ in the level of their output target. Specifically, as assumed above the central bank targets the potential output level whereas the government aims at a higher level. The analysis implies that our payoffs $\{a, b, c, d, w, x, y, z\}$ depend on two broad factors.

First, it is the policymakers' (and society's) costs of output variability relative to inflation variability. These in turn depend on the structure of the economy and the extent of various rigidities present at the micro-foundations level. For example, a greater rigidity in price and/or wage setting will increase this cost in most models. Second, it is the relative weights assigned to inflation and output stabilization in the policy loss function (the degree of conservatism of the two policymakers), as well as the degree of the government's ambition in stimulating output. In the real world these are functions of various political economy or structural factors mentioned above.

The discussion implies that these payoffs are difficult to calibrate in a way encompassing different underlying macroeconomic models. We believe that reasonable per-period values of the conflict cost relative to the victory gain for the central bank lie in the interval $\frac{a-b}{a-d} \in \left[\frac{3}{2}, 3\right]$, but report the threshold in Figure 7 for a larger interval, and for $\alpha \in \left\{\frac{1}{2}, 1, 2\right\}$. The $\mathcal{M}$ commitment values r above the solid lines ensure the ECB's achievement of the inflation target on average, and discipline the member governments. The r values below the solid lines are likely to be insufficient to achieve that as they lead to multiple SPNE. The calibration implies the following tentative conclusion.

Remark 1. *Given the degree of $\mathcal{F}$ rigidity and ambition of the EMU countries implied by their past outcomes, the required degree (explicitness) of the ECB's long-run commitment to low inflation may be substantial.*

Specifically, such commitment should be explicit enough for all parties to believe that it will not be 'reconsidered' for at least 3-5 years, but more likely significantly longer.[29]

Obviously, a monetary union is subject to a possible *free riding* by individual governments. It can be argued that the potential benefits of excessive $\mathcal{F}$ policy accrue primarily in the indisciplined country, whereas the punishment by the common central bank in the form of higher interest rates is spread across all the member countries. Therefore, if member countries do not internalize the negative externality cost they impose on other

[29]In stochastic terms, the percieved probability $\theta_{\mathcal{M}}$ that the central bank will not be able to 'reconsider' its (explicit or implicit) preferred average inflation level at its monthly meeting has to be (substantially) less than 2-3%.

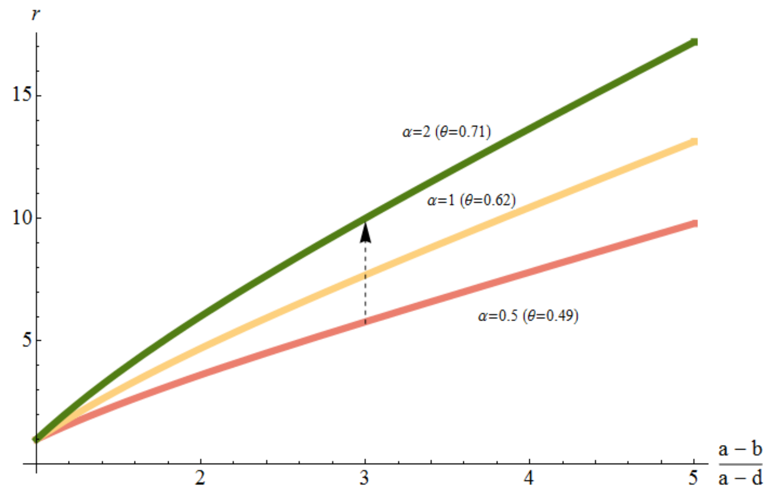

FIGURE 7. Dependence of the $\mathcal{M}$ commitment threshold r on the ECB's conflict cost relative to the victory gain, $\frac{a-b}{a-d}$, using (20) and varying the value of α.

members, a stronger punishment may be required to discipline them compared to a single country setting.

Such free riding can be modeled in our framework by increasing the conflict cost of the central bank $(a-b)$. The bank now has to fight the government harder, and hence it suffers a greater disutility from doing so. As one would expect, (20) shows that an even stronger $\mathcal{M}$ commitment will be required to discipline governments under such free riding.

It may in some models be the case that if the free riding problem is sufficiently severe some individual governments' best response to L may be h, ie we have $x > w$ similarly to the Chicken scenario. Our analysis implies that in such case, reminiscent of the situation in Greece and some other member countries, even an infinitely strong $\mathcal{M}$ commitment cannot discipline such governments. The same is true, even without free riding, in the case of a very high $\mathcal{F}$ ambition. If $x > w$ and $z > y$ then l is a strictly dominated strategy in the static game, and hence no amount of $\mathcal{M}$ commitment can possibly make the government(s) discipline their actions.[30]

7. Extensions

7.1. **Combinations of Probability Distributions Using Mean Values.** This section reports a statistical result which allows us, in some cases, to write the above necessary and sufficient conditions in a more elegant fashion. Specifically, it is done using solely the *mean value* of the underlying probability distribution, without reference to its other moments. This also means that we can obtain analytical solutions for combinations of distributions that are very *different* in nature (unlike in Cases 1-3 in which all the $\mathcal{F}$ revisers within each case had the same type of distribution).

[30]For this reason, our framework does not offer a tool to escape inefficient equilibria in some classes of game, eg the Prisoner's Dilemma.

Let us denote μ_n to be the mean value of the underlying probability distribution of the n'th reviser. The following is a known result in statistics, see eg Lemma 2.4 in Kallenberg (2002).

Lemma 1. *Consider $F(t)$ from Definition 3 such that*

$$F(r) = 1. \tag{22}$$

Then

$$\int_0^r (1 - F(t))\,\mathrm{d}t = \mu. \tag{23}$$

Let us mention the interpretation of (22): it ensures that (all) reviser(s) have the opportunity to make a revision. Lemma 1 implies that, if (22) holds, even probability distributions expressing very complicated timing of moves can be 'summarized' without loss of generality by their first moments. Put differently, if (22) is satisfied then μ_n fully describes the degree of commitment/rigidity of each reviser.

The following result uses Lemma 1 to express Proposition 1 in an alternative fashion. Note however that while it is easier to use in combining different probability distributions, it is not as general as Proposition 1 since (22) is required to hold for every underlying distribution.[31]

Proposition 3. *Consider the dynamic r-period game policy interaction described by either (1), (2), or (3), whereby $\mathcal{F}$ rigidity of each member n is described by an arbitrary probability distribution with a mean value of μ_n. Under $F_n(r) = 1, \forall n$, the necessary and sufficient condition for the committed player to win the game, (7), can be written as*

$$\frac{r}{\sum_{n=1}^{N} w_n \mu_n} > \bar{R}. \tag{24}$$

Proof. Substituting Lemma 1 into (19) yields (24). □

To demonstrate the usefulness of this 'shortcut', let us report an example that combines the above Case 2 with normally distributed moves.

Example 1. *Consider a monetary union consisting of two equally sized member countries, whose $\mathcal{F}$ policymakers' timing of moves has the following form:*

- *Country 1: uniformly distributed moves of Case 2, $F_n(t)$ from (12),*
- *Country 2: normally distributed moves, such that*

$$F_2(t) = \frac{\Phi_{\mu_2,\sigma^2}(t)}{\Phi_{\mu_2,\sigma^2}(r) - \Phi_{\mu_2,\sigma^2}(0)} \tag{25}$$

where

$$\Phi_{\mu_2,\sigma^2}(t) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{t} e^{-\frac{(x-\mu_2)^2}{2\sigma^2}}\,\mathrm{d}x,$$

[31] For example the functions $F(t)$ depicted in Figures 2 and 5 do *not* satisfy the condition, and hence the following result is not applicable to them.

is the CDF of a normal distribution (truncated on the interval $[0, r]$*), and where* μ_2 *and* σ *are its mean and standard deviation (and* $x \in \mathbb{R}$*). Then the necessary and sufficient degree of* $\mathcal{M}$ *commitment for* $\mathcal{M}$ *to win the game is*

$$\frac{r}{\mu_1 + \mu_2} > \bar{R}, \tag{26}$$

where $\mu_1 = \frac{g+h}{2}$ *is the mean value of the probability distribution of Case 2.*

For a graphical depiction see Figure 8. Let us note two things. First, the condition (22) is satisfied for both countries. Second, the standard deviation σ does not determine the threshold value of r.

7.2. **Discounting.** It is apparent that discounting by the *reviser* has neither qualitative nor quantitative effect on the outcomes of the r-period dynamic game. This is because the Revision is a static best response to the observed action by the committed player. This section shows that while discounting by the committed player himself does have an effect, it is only a quantitative one. Specifically, the committed player's impatience works in the predicted direction of making it harder to coordinate and win the game.[32]

Proposition 4. *Consider the dynamic* r*-period game of policy interaction described by either (1), (2), or (3), in which the committed player discounts the future by* $e^{-\rho t}$*, where* $\rho \in [0, \infty)$*. The necessary and sufficient degree of* $\mathcal{M}$*'s relative commitment to win the game is*

$$\frac{\int_0^r e^{-\rho t} \mathrm{d}t}{\int_0^r e^{-\rho t} \left(1 - F(t)\right) \mathrm{d}t} > \bar{R}, \tag{27}$$

ie its strength is increasing in the degree of his discounting (impatience), ρ*.*

Proof. The proof of this statement follows from the fact that $(1 - F(t))$ is a decreasing function. Therefore, an increase in ρ decreases more than proportionally the integral in the numerator than the integral in the denominator. In other words, the increase of ρ decreases the fraction on LHS. Consequently, in order to achieve the required value of relative commitment $\bar{R}$ for greater ρ, the value of r must increase. ☐

The thresholds $\bar{R}$ are again identical to those reported in Proposition 1. If we interpret, similarly to the literature, ρ as a decreasing function of the central banker's *goal*-independence, the proposition implies its *substitutability* with explicit inflation targeting. For empirical evidence of this relationship see Libich (2008).[33] The following result summarizes the effects of discounting.

Corollary 1. *There exists* $\bar{\rho} > 0$ *such that: (i) for all* $\rho < \bar{\rho}(\bar{R})$ *an* r *value satisfying (27) exists, whereas (ii) for all* $\rho \geq \bar{\rho}$ *even an infinitely strong commitment* $r \to \infty$ *does not satisfy the condition.*

[32] The analysis of the committed player's discounting can be made more parsimonious by incorporating it into the function $F(t)$. We however do not do so in order to keep the intuition of $F(t)$ as a Revision function.

[33] Let us note that the Debelle and Fischer (1994) distinction between instrument and goal independence is important here, since the former is a *complement* (in fact a pre-requisite) of explicit inflation targeting.

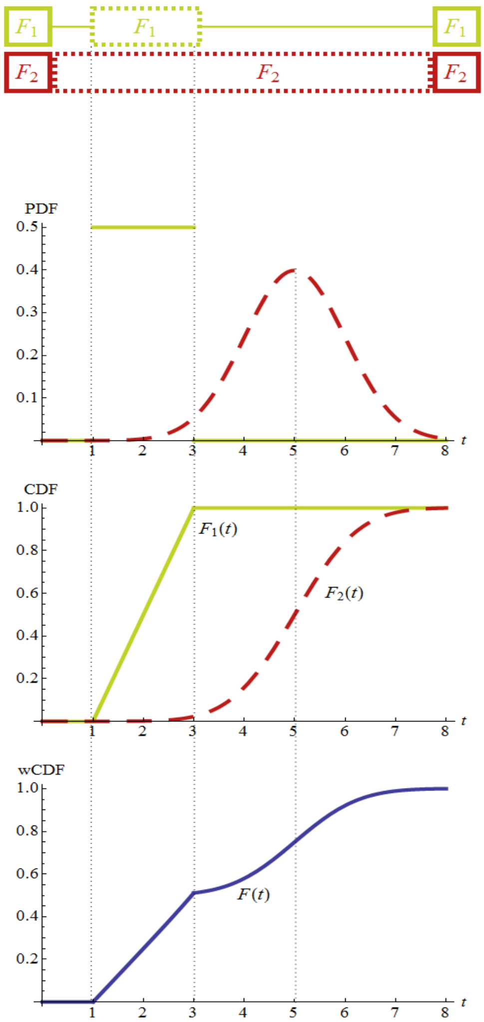

FIGURE 8. The PDFs, CDFs, and wCDF of Example 1 featuring $r = 8$ and two equally weighted revisers with the following Revision functions: uniformly distributed on $[1, 3]$ ($\mathcal{F}$ policymaker 1), and truncated normally distributed with $\mu = 5$ and $\sigma^2 = 1$ ($\mathcal{F}$ policymaker 2).

Proof. For the sake of brevity, we perform the proof for continuous distributions $F(t)$ - discrete are analogous. Then there exists $p > 1$ such that $F(p) = q$ with $\frac{1}{1-q} < \bar{R}$. We

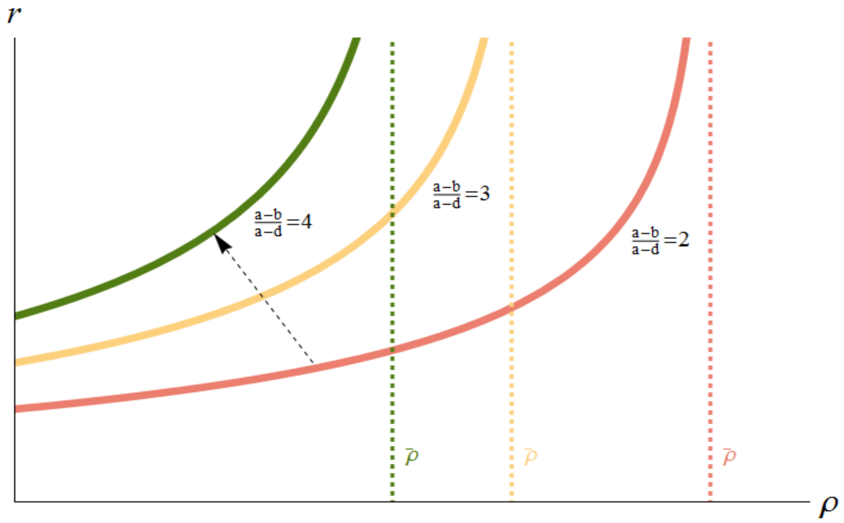

FIGURE 9. The threshold of r as a function of the discount factor ρ under three $\bar{R}$ (and showing $\overline{\rho}$ values in each case).

can therefore express the value of the LHS of (27) in the following way:

$$\begin{aligned}\frac{\int_0^\infty e^{-\rho t}dt}{\int_0^\infty e^{-\rho t}(1-F(t))dt} &= \frac{\int_0^p e^{-\rho t}dt + \epsilon(\rho)}{\int_0^p e^{-\rho t}(1-F(t))dt + \delta(\rho)} \\ &< \frac{\int_0^p e^{-\rho t}dt + \epsilon(\rho)}{(1-q)\int_0^p e^{-\rho t}dt + \delta(\rho)} \\ &\xrightarrow{\rho\to\infty} \frac{1}{1-q} \\ &< \bar{R},\end{aligned}$$

where $\epsilon(\rho)$ and $\delta(\rho)$ approach zero as $\rho \to \infty$. This, in combination with the monotone dependence of the required r on ρ (Proposition 4), competes the proof. □

Figure 9 plots the necessary and sufficient threshold of r as a function of various $\bar{R}$ and the discount factor ρ, indicating the $\overline{\rho}$ values for each $\bar{R}$. The r values above the curves satisfy the condition (27).

The first implication of this section is that our above findings are robust to discounting, as full patience of the committed player is not necessary for his win. Nevertheless, the second observation is at odds with the outcome under standard commitment, in which Stackelberg leadership delivers a win to $\mathcal{M}$ regardless of his discount factor. In our dynamic setting, if the committed player is sufficiently impatient, $\rho > \hat{\rho}$, even an infinitely strong $\mathcal{M}$ commitment falls short of securing his win. This suggests that the static nature of the standard commitment concept can be a serious shortcoming. As most macroeconomic games are dynamic in nature, caution should be exercised in relying heavily on the results of the static commitment concept.

7.3. **Repetition.** Let us extend the analysis and allow for a longer horizon and a further (possibly infinite) repetition of the r-period dynamic game we have studied so far. It follows from Assumption 1 that the full repeated game consists of a sequence of potentially different r-period dynamic games. Let us therefore introduce $F_n^m(t)$, where $m \in \mathbb{N}$, as an individual Revision function following the m'th simultaneous move of the n'th reviser.

Combining the findings of Sections 7.2-7.3 with those of Section 4 proves the following generalization of Proposition 1.

Theorem 1. *Consider the game described by either (1), (2), or (3) with any number of simultaneous moves and an arbitrary timing of Revisions summarized by $F_n^m(t)$. The committed player wins the game if and only if*

$$\frac{\int_0^r e^{-\rho t} \mathrm{d}t}{\int_0^r e^{-\rho t}\left(1 - F^m(t)\right) \mathrm{d}t} > \bar{R}, \tag{28}$$

where the thresholds $\bar{R}$ are as reported in Proposition 1.

The only difference relative to (27) is the fact that if repetition is allowed the above derived necessary and sufficient condition has to hold for each and every of the r-period part of the games. If this is not the case then there will be additional SPNE also featuring the H and/or h levels.

This result nests the special case in which the timing of the Revision is the same for an individual player across all r-period dynamic games, ie $F_n^m(t) = F_n(t), \forall m$. In such case the r-period dynamic game is a *dynamic stage game*, and the whole game is a *repeated dynamic game*. Intuitively, if the dynamic stage game has a unique SPNE then we know that the effective minimax values (the infima of the players' subgame perfect equilibrium payoffs (Wen (1994)) are the payoffs that obtain from that SPNE. If this unique SPNE is Pareto-efficient, then the effective minimax values of the repeated game will be equivalent to those of the dynamic stage game - since these cannot be improved upon. Put differently, since the outcome lies on the Pareto frontier the set of Pareto superior payoffs is empty.[34]

7.4. **Endogenous Timing of Moves.** It is straightforward to endogenize the degrees of commitment in our framework. One can include a per-period net-cost, which will summarize all the (unmodelled) costs and benefits of moving less frequently, and let the players choose their timing optimally at the beginning of the game.[35]

The players may then face a trade-off; a greater commitment may achieve their preferred outcome, but it may be costly. Therefore, whether or not a player commits, and to what extent he does, will be a function of various variables describing the game. In terms of the policy interaction, if there is no cost involved in long-term committing then the central bank will choose an r level such that its commitment is (well) above the threshold. If however committing is sufficiently costly that the central bank may not commit.

[34] The result is consistent with the body of literature showing that the Folk Theorem may not apply in some asynchronous games, see eg Takahashi and Wen (2003).

[35] This is similar to Bhaskar (2002) who considers a simple way to endogenously determine Stackelberg leadership.

This is demonstrated in Libich and Stehlík (2009) in a different (New Keynesian) setting without $\mathcal{F}$ policy, where the optimal degree of long-run $\mathcal{M}$ commitment r^* to eliminate the time-inconsistency problem is shown to be a function of the structure of the economy, the frequency with which agents update expectations, and also the potential short-run costs in terms of stabilization inflexibility, and the benefit of better anchored expectations.

8. Summary and Conclusions

The paper models the interaction between fiscal ($\mathcal{F}$) and monetary ($\mathcal{M}$) policy - in a monetary union as well as in a single country setting. The aim is to consider under what circumstances, if any, excessive $\mathcal{F}$ policies can undermine the credibility and outcomes of $\mathcal{M}$ policy, and whether the design of $\mathcal{M}$ policy can indirectly induce a change in the undesirable $\mathcal{F}$ stance.

The paper's main contribution lies in examining the interaction of $\mathcal{M}$ policy and (any number of) $\mathcal{F}$ policies in a novel game theoretic setting, in which the timing of the policies' actions is no longer repeated every period in a simultaneous fashion. Our framework is general enough to allow for an arbitrary probability distribution of the policymakers' moves (both deterministic and stochastic), as well as an arbitrary combinations of probability distributions. For illustration we complement the results for a general setting by depicting several realistic scenarios, namely uniform, normal, and binomial distributions, that latter in line with the Calvo (1983) timing.

All settings show that if the central bank is sufficiently strongly (explicitly) committed in the long-term, it can resist $\mathcal{F}$ pressure and ensure the credibility of $\mathcal{M}$ policy and stable prices. Furthermore, unless $\mathcal{F}$ ambition is very high, or there exists a significant free riding problem in the $\mathcal{M}$ union, such strong $\mathcal{M}$ commitment has the potential of *disciplining* $\mathcal{F}$ policies. It does so by reducing the incentives of governments from excessive policies, and hence it improves the policy coordination and outcomes of both policies.[36]

All settings however also show that if $\mathcal{M}$ commitment is insufficient than $\mathcal{F}$ accesses may spill over and cause undesirable $\mathcal{M}$ outcomes with excessive inflation. In such a situation significant macroeconomic imbalances may built up over time with adverse consequences. In order to better understand how much $\mathcal{M}$ commitment is required to avoid such situations, we show the threshold degree to be an increasing function of: (i) $\mathcal{F}$ rigidity and ambition of the member countries, (ii) their relative economic size, (iii) the cost of a policy conflict - relative to the gain of improvement in the policy coordination and long-term outcomes (affected by various deep parameters of the underlying macroeconomic model), and (iv) the central banker's impatience.

The latter implies that a less patient central bank needs to commit more strongly (explicitly) to ensure its credibility. Interpreting patience as an increasing function of the

[36]Let us mention that while long-term $\mathcal{M}$ commitment has usually been specified in terms of an inflation target for consumer prices, our commitment concept is not limited to such a specification. Put differently, we do not impose a concrete *type* of $\mathcal{M}$ commitment to be pursued - our analysis reports the *degree* of $\mathcal{M}$ commitment required in the face of ambitious and rigid $\mathcal{F}$ policies. This is an advantage since the global financial crisis of 2007-9 brings back to the fore the question of whether central banks should respond to a broader measure of inflation, potentially also including various asset prices (which the existing literature commonly answered in the negative, eg Bernanke and Gertler (2001)).

degree of central bank goal-independence, this offers an explanation for the fact that inflation targets were more explicitly grounded in countries originally lacking central bank goal-independence (such as New Zealand, UK, Canada, and Australia) than in those with a rather independent central bank (such as the US, Germany, and Switzerland).

An inportant insight that can be modelled formally in our framework is the free riding problem in a monetary union. But running excessive $\mathcal{F}$ policy an individual member country imposes a negative externality on the rest of the union in the form of higher average interest rates. If this externality is not internalized each member country has an extra incentive to be spend excessively. It can be argued that this reasoning can be applied to the current debt crisis of Greece and some other EMU members.

Our findings are related to several existing literatures. First, our commitment concept is compatible with the timeless perspective commitment postulated by Woodford (1999) and frequently used since then. This is because our long-run notion of commitment does not place any restrictions on how stabilization policy should be conducted, ie how the short term (interest rate) policy instrument should be adjusted in response to shocks. In fact, our commitment does not even restrict *how* long-term decisions about the policy stance should be made, it only puts a constraint on *how often* they can be made. This also implies that if (and only if) the objective of $\mathcal{M}$ is postulated as a long-run goal - achievable on average of the business cycle, it does *not* require the central bank to become more conservative (strict) in achieving it, and does not compromise the flexibility in stabilizing the real economy in response to shocks (as in Rogoff (1985)).[37]

Second, the work of Schaumburg and Tambalotti (2007) also examines the gains from $\mathcal{M}$ commitment (which they call quasi commitment as it lies anywhere between discretion and timeless perspective commitment). Similarly to our paper, the authors find that a stronger commitment leads to an improvement in $\mathcal{M}$ policy credibility and outcomes.[38] Their analysis however does not include $\mathcal{F}$ policy, and hence it is commitment in absolute terms. In contrast, our analysis highlights the fact that it is $\mathcal{M}$ commitment relative to $\mathcal{F}$ rigidity and ambition that matters.

Third, there exists a large empirical literature on the effects of explicit inflation targeting. While the findings are far from conclusive, there exists fair support for most of our results. Among other, explicit inflation targets have been shown to reduce the nominal interest rate (and hence inflation) and its volatility to a larger extent than non-IT countries (eg Siklos (2004), Neumann and von Hagen (2002)), without an increase in output volatility (eg Corbo, Landerretche and Schmidt-Hebbel (2001)), Arestis, Caporale and Cipollini (2002), Fatas, Mihov and Rose (2004)). In terms of the disciplining

[37]It should be noted that explicit inflation targets in almost all industrial countries have indeed been specified in such a medium/long-run fashion, see eg Mishkin and Schmidt-Hebbel (2001). As Svensson (2009) argues: *'Previously, flexible inflation targeting has often been described as having a fixed horizon, such as two years, at which the inflation target should be achieved. However, as is now generally understood, under optimal stabilization of inflation and the real economy there is no such fixed horizon at which inflation goes to target or resource utilization goes to normal.'* Obviously, as a temporary measure some transition countries may opt for a short-run specification of the targeting horizon after adoption in order to build up the credibility of the target.

[38]A different avenue with similar conclusions is pursued by Orphanides and Williams (2005), and informally such arguments have been made by eg Bernanke (2003), Goodfriend (2003), and Mishkin (2004).

effect of $\mathcal{M}$ commitment on $\mathcal{F}$ policy, the preliminary results of Franta et al. (2009) using an estimated DSGE model featuring the degree of $\mathcal{F}$ dominance confirm the above findings.[39]

Nevertheless, more research is required to assess whether, and under what circumstances, a move towards institutionalizing a stronger $\mathcal{M}$ commitment of a long-term nature does indeed translate into an improvement in the long-term stance of $\mathcal{F}$ policy. In doing so, the issue of causality vs correlation has to be carefully examined, since both a stronger $\mathcal{M}$ commitment, and an improvement in $\mathcal{F}$ policy may be driven by an underlying common factor.

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[39] This is also consistent with the case study by Don Brash quoted in the introduction.

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Appendix A. EMU Data

We use the data from Eurostat to create our variables S, θ, and w for each member country n, reported in the following Table. The way these are created is described in the main text.[40] While the individual country values in the table are rounded to two or three decimal places, the Eurozone averages as well as the calculations in the main text have been done with nine decimal places.

Country n	Weight w_n	Surplus S_n	$\mathcal{F}$ Rigidity θ_n $(\alpha = 1)$
Austria	0.033	-1.05	0.51
Belgium	0.039	-0.22	0.18
Cyprus	0.001	-3.47	0.78
Finland	0.021	3.4	0
France	0.218	-2.95	0.75
Germany	0.327	-3.22	0.76
Greece	0.019	-5.04	0.83
Ireland	0.015	1.05	0
Italy	0.147	-3.6	0.78
Luxembourg	0.004	1.35	0
Malta	0.001	-5.38	0.84
The Netherlands	0.061	-1.12	0.53
Portugal	0.016	-3.92	0.8
Slovenia	0.003	-2.37	0.7
Spain	0.094	0.2	0
Average (non-weighted)	0.067	-1.75	0.5
Average (weighted)		-2.4	0.62

[40] The only exception is Greece, for which the 2001 budget balance is not stated in the Eurostat database, and hence the 2002-2006 period is averaged over to create S.