Stochastic Timing, Uniqueness, and Efficiency in Games[1]

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Abstract

In existing game theoretic settings the timing of moves is deterministic, i.e. they occur with certainty at a pre-specified time. To add more realism we propose a framework in which, after an initial simultaneous move in time $t = 0$, one player gets to revise his action with positive probability at some $t \geq 0$. Since the initial action of the opponent can be observed, and payoffs accrue over time, the setup constitutes a dynamic extension of the Stackelberg leadership concept. Allowing for an arbitrary timing distribution, and using both subgame perfection and stochastic stability, we derive the necessary and sufficient conditions under which our dynamic revision game has a unique efficient outcome even if the underlying normal form game has no efficient Nash, or multiple ones. Intuitively, the fact that a player is less likely to move than the opponent may serve as a commitment device. Therefore, if the revision opportunity is expected to arrive sufficiently early then the committed player's initial cost of mis-coordination or conflict will be more than compensated by ensuring his preferred outcome after the opponent's revision. The framework allows, among other things, to address the issue of equilibrium selection in games in which traditional equilibrium selection approaches fail such as the Battle of the Sexes and the Game of Chicken. It also offers some insigths into the debate about Pareto-dominance versus risk-dominance.

Keywords: asynchronous moves, stochastic timing, equilibrium selection, revision, asymmetric coordination games.

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1. Introduction

Many normal form games feature an inefficient outcome in equilibrium. It can be either the unique Nash equilibrium (e.g. in the Prisoner's Dilemma or the Dynamic Inconsistency game), or coexist with one or more efficient Nash equilibria (e.g. in coordination and anti-coordination games). In an attempt to address this issue the game theoretic literature introduced various realistic aspects of the real world that the stylized normal form game disregards, either in the form of repetition, equilibrium refinements, or by slightly changing the game.[5]

We follow the latter avenue by enriching the timing structure of the game, and hence the strategy space of the players. Specifically, after an initial simultaneous move at (continuously measured) time $t = 0$, one of the players (called *reviser*) can revise his strategy before the end of the game, $t = 1$; observing the action of the *committed player* who cannot do so. We allow for an arbitrary probability distribution of such revision opportunity, both deterministic and stochastic.

In the real world, the inability to move frequently, which acts as a commitment device, may be due to various technological, logistical, and behavioural constraints. For example, if a firm has to use a marten oven for its production process and it is not switched on, it will take some minimal time before it becomes operational. Letting the rival observe the current status of the marten oven allows the firm to commit to not changing the quantity of production, or perhaps even not entering the market before a certain date. Another source of such commitment is an exogenous meetings schedule of the management within organizations that determines the timing of decisions. In terms of macroeconomic examples, various forms of rigidity (in prices, wages, and information) have been investigated and found empirically relevant. All these examples show that the timing of many real world games may differ from the stylized simultaneous move game, and that this may affect the outcomes.

Our setting can be thought of as a more general, dynamic version of the Stackelberg leadership (commitment) concept since the payoffs accrue continuously over $t \in [0, 1]$. We show that such extension produces novel insights. The standard Stackelberg leadership (commitment) concept delivers a unique and efficient subgame perfect equilibrium (SPE) in all the above mentioned classes of games except the Prisoner's Dilemma *regardless* of the exact payoffs and players' discount factors. In contrast, we derive the necessary and sufficient conditions (for each game) under which this happens in our dynamic framework. They are a function of the payoffs, degree of discounting, and most importantly the timing of the revision opportunity. Intuitively, the expected revision time must be sufficiently short in order to for the committed player's gain from achieving its preferred outcome more than offset the potential cost of mis-coordination or conflict. Put differently, the committed player's degree of commitment has to be sufficiently high relative to the reviser's.

Our main attention lies in normal form games that have two pure strategy Nash equilibria mapping into each as one changes the names of the players. Examples of such

[5] The Folk Theorem (first proved in general case by Rubinstein (1979)) points out to repetition as a way to enforce an efficient outcome in a game. Equilibrium refinements such as perfection Selten (1975) allow to avoid equilibria in weakly dominated strategies in coordination games. Examples of a slight change in the game are Farrell (1987), Shaffer (2004), and Basov, Smirnov, and Wait (2007).

games are the Battle of the Sexes and the Game of Chicken that arise naturally in many areas of economics as well as other disciplines such as biology.[6] Unlike standard equilibrium selection techniques that cannot select a unique equilibrium, in our framework the positive ex-ante revision probability can do so even if the reviser ends up not being able to move ex-post.

Another case of interest is the Dynamic Inconsistency game often used to describe the problems of monetary policy and high inflation of the 1970s (for details see Kydland and Prescott (1977)). By offering the central bank the chance to commit more than the public, the unique inefficient equilibrium of the normal form game can replaced by the efficient SPE in the set of equilibrium outcomes of the dynamic revision game. Obviously, this is not the case in games such as the Prisoner's dilemma, in which both players have a strictly dominant strategy in the normal form game. This is because commitment does not constitute an advantage in such games.

Having derived the necessary and sufficient conditions for a unique efficient equilibrium by subgame perfection, we then consider what happens if these are not satisfied, i.e. if the reviser is insufficiently flexible relative to the committed player. In doing so we use evolutionary techniques developed by Foster and Young (1990), Kandori, Mailath, and Rob (1993), and Young (1993). We show that even if the expected time of revision opportunity is $t \to 1$, that is for an arbitrary timing distribution, an efficient outcome (preferred by the committed player) is uniquely selected. It should be noted that in some coordination games, e.g. the Stag Hunt, this result is in stark contrast to the literature whereby evolutionary techniques generally favour the risk-dominant equilibrium over the Pareto dominant equilibrium, for a review of this literature see Ellison (2000).

The paper is organized in the following way. Section 2 presents the framework. Section 3 reports results for several classes of games using subgame perfection. In Section 4 we revisit those results using stochastic stability. Section 5 concludes.

2. The Setup

Consider a 2×2 simultaneous move game of the following general form:

(1)

	l	h
L	a, w	b, x
H	c, y	d, z

Let us construct a dynamic revision game in the following way. We assume that the players move simultaneously at the beginning of the game, at time $t = 0$, by choosing one of the actions in the simultaneous move game shown in (1). The column (*committed*) player, denoted by C, cannot move again and hence has to stick to his initial choice to the end of the game (normalized to $t = 1$). In contrast the row player (*reviser*), denoted

[6] For instance, consider a situation from the theory of industrial organization when two firms consider entering one of the two markets. Each firm prefers the same market provided it enters it alone, however, they both prefer being in a different market to competing. Another example arises in macroeconomics when the fiscal and monetary authorities have different objectives. They both prefer coordination to conflict, but they each favour a different pure Nash (the government one with higher spending that boosts output and leads to higher inflation, whereas the central bank one with lower spending and hence lower output and inflation as in Nordhaus (1994).

by R, can move again with some positive probability, observing the initial play of the committed player.

We will allow for an arbitrary probability distribution on the timing of the reviser's response. Formally, will describe the timing of the revision opportunity by the following:

Definition 1. *A* ***revision function*** *is a cumulative distribution function (CDF), denoted $F(\cdot)$, expressing the probability that the reviser has had a revision opportunity* ***before*** *time t, that is*

$$F(t) = P(\tau < t), \tag{2}$$

where τ is the ***actual revision time*** *of the reviser.*[7] *Her* ***expected revision time*** *will be denoted by μ.*

For three examples of $F(t)$ see Figure 1. We will consider functions such that $F(1) \leq 1$. Allowing for cases of $F(1) < 1$, i.e. scenarios in which the reviser may not be able to respond (from an ex-post perspective) *before* the end of the game is for generality. Nevertheless, since the payoff functions will be unaffected by the play in the closing time $t = 1$, we will for modelling convenience assume that $F(1+) = 1$. In words, if $F(1) < 1$ then there will be a (payoff irrelevant) jump in the CDF to 1 (see the middle panel of Figure 1) This technical assumption will make sure that the actual revision time τ lies in the closed interval $[0, 1]$, and hence the expected revision time μ is always defined.[8]

Given the revision function $F(t)$ one can define the *reaction speed* of the reviser as $RS = \int_0^1 F(t)dt$. Similarly, her *degree of commitment* (both in absolute terms and relative to the committed player) is $DC = \int_0^1 (1 - F(t))dt$, commonly called the *complementary CDF*. Let us note two things. First, our setting nests the standard simultaneous move game in which $RS = 0$, as well as the Stackelberg leadership game in which $RS = 1$. Second, if $F(1) = 1$ then $\mu = DC$.

Let us describe the strategies and the players' payoffs in the dynamic revision game. Since we assume that the committed player whas to stick to his initial choice up to the end of the game his strategy space is the usual $S_C = \{l, h\}$. The reviser's strategy space is however enlarged by the time dimension, namely

$$S_R = \{L, H\} \times \mathcal{F},$$

where $\mathcal{F}$ is a set of functions from $\{l, h\} \times [0, 1]$ into $\{L, H\}$. Any function $f \in \mathcal{F}$ prescribes the choice of the reviser as a function of the observed choice of the committed

[7]Let us mention that there exists an alternative specification of CDF in the probability theory literature that also includes time t, that is $F(t) = P(\tau \leq t)$. We opt for the specification of (2), without loss of generality, as it simplifies the exposition. Let us also mention that the reviser can potentially have more than one revision opportunity on $t \in [0, 1]$, but since the second and further revision opportunities occur under the same circumstances to the first, one can focus on the latter.

[8]Note that since the action in $t = 1$ is payoff irrelevant, the reviser is indifferent between revising and not revising at that time regardless of the committed player's action. Libich and Stehlík (2008) do not make this 'jump' assumption in order to be able to consider a repeated version of this game in which the players make, in addition to stochastic moves, a simultaneous move with certainty every $r \in \mathbb{N}$ periods. For this reason, they use the $F(t) = P(\tau \leq t)$ specification of CDF instead of (2).

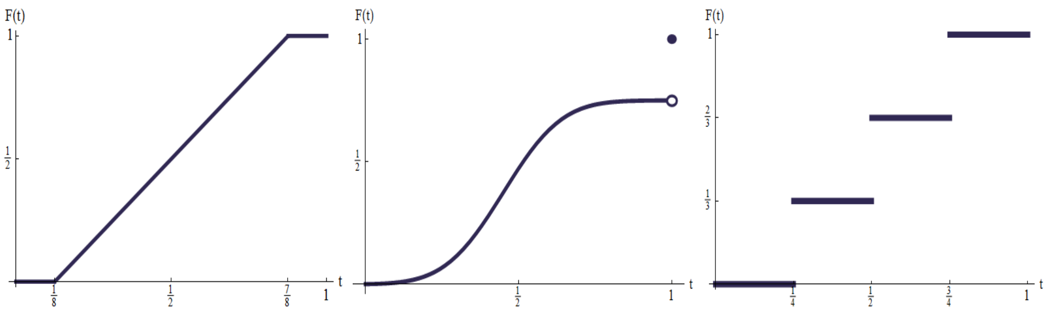

FIGURE 1. Three examples of the CDF, $F(t)$, with uniformally, normally, and discretely distributed moves respectively (each with $\mu = \frac{1}{2}$).

player, *and* the time at which the revision opportunity arrives. The players' payoffs are:

$$\begin{aligned} u_R(s_R, s_C) &= E[\tau U_R(X,Y) + (1-\tau)U_R(f(Y,\tau),Y)], \\ u_C(s_R, s_C) &= E[\tau U_C(X,Y) + (1-\tau)U_C(f(Y,\tau),Y)], \end{aligned} \tag{3}$$

where $s_R = (X, f)$, $X \in \{L, H\}, s_C = Y \in \{l, h\}$, and the expectation is taken with respect to the distribution of the random revision time τ. It is now apparent that while the timing of the revision opportunity affects the players' payoffs and hence the initial simultaneous move, it does not affect the revision decision. Formally:

Lemma 1. *Let (X, f), where $X \in \{L, H\}$ and $f \in \digamma$, be the equilibrium strategy of the reviser. Then $f(\cdot, \tau) = f(\cdot)$, i.e. the decision to change her previous action depends only on the observed choice of the committed player, not on the time τ the revision opportunity arises.*

Proof. Consider a revision opportunity arriving at $\tau < 1$. The reviser will choose some strategy $Z = \{L, H\} \backslash \{X\}$ if and only if

$$U_R(Z, Y) > U_R(X, Y). \tag{4}$$

That is, we have $f(Y, \tau) = Z$ if (4) holds, and $f(Y, \tau) = X$ otherwise, irrespective of τ. □

Throughout the paper our aim is to derive, for each considered game, the necessary and sufficient conditions for the dynamic revision game to have a unique and efficient equilibrium outcome. In doing so Section 3 uses subgame perfection and Section 4 uses stochastic stability. In relation to the former solution concept, the following terminology will be useful:

Definition 2. *The committed player will be said to* ***win the game*** *iff the dynamic revision game has a unique SPE in which his payoff corresponds to that of his preferred (highest payoff) outcome of the underlying normal form game.*

3. Subgame Perfection

We will formulate our main result using the Battle of the Sexes game in the next section, but Sections 3.2-3.4 show that the result applies to the Game of Chicken, Pure Coordination game, Stag and Hunt, and the Dynamic Inconsistency game as well. The only difference is quantitative in regards to the magnitude of the necessary and sufficient threshold of the expected revision time $\bar{\mu}$.

3.1. **The Battle of the Sexes.** The payoffs of this class of game satisfy:

$$a > d > \max\{b, c\} \quad \text{and} \quad z > w > \max\{x, y\}. \tag{5}$$

There are two pure strategy Nash equilibria, (L, l) and (H, h), and one in mixed strategies, where the row player plays L with probability $p = (z - y)/(w - y + z - x)$ and the column player plays l with probability $q = (d - b)/(a - b + d - c)$.

As the pure strategy equilibria are asymmetric with the row player preferring (L, l) and the column player preferring (H, h), the symmetric mixed strategy equilibrium is often conjectured to be played. However, it is dynamically unstable: should the row player, for example, deviate slightly from her equilibrium probability in favour of (L, l), it will become the best response of the column player, reinforcing the initial deviation of the row player. Therefore, one would expect the players to eventually end up playing of the pure strategy equilibria.[9]

Note that if $a = z$, $w = d$, $b = y = x = c$ the simultaneous move game is symmetric with respect to renaming the players and the following transformation $L \leftrightarrow H$, $l \leftrightarrow h$. Therefore, standard anonymous equilibrium selection technique cannot be applied to select an equilibrium. In our framework this is not the case. The symmetry is broken by allowing the revision opportunities to differ across the players.

For illustration, we will also be reporting outcomes for a specific (and symmetric) set of payoffs satisfying (5), namely

(6)

	l	h
L	$1, \frac{2}{3}$	$0, 0$
H	$0, 0$	$\frac{2}{3}, 1$

Let us now report the main result of the paper.

Theorem 1. *Consider the Battle of the Sexes game described by (5), and an arbitrary timing of the revision opportunity summarized by $F(t)$. The necessary and sufficient condition for the committed player to 'win the game' is*

$$\mu < \bar{\mu} < 1, \tag{7}$$

where the threshold expected revision time $\bar{\mu}$ is

$$\bar{\mu} = \frac{z - w}{z - x} \overset{(6)}{=} \frac{1}{3}.^{10} \tag{8}$$

Proof. In any SPE the behavior of the reviser at the time of the revision opportunity is described in Lemma 1. Therefore, we can focus on the players' choices in $t = 0$. If the initial choice of the committed player is h then the reviser's best response is H at $t = 0$ and never revise. Therefore (H^*, h) is a SPE, where we define H^* as

$$H^* = s_R^* = (H, f(h, \tau) = H, f(l, \tau) = L), \tag{9}$$

[9] Another reason to settle on a pure strategy equilibrium is that the utility of each player - even in the less preferred pure equilibrium - is higher than in the mixed strategy one.

[10] The right hand side of the $\overset{(6)}{=}$ notation will throughout report the condition for the specific payoffs, in this case (6).

For (H^*, h) to be the *unique* SPE, i.e. for the committed player to win the game (see Definition 2), he should find it strictly optimal to choose h even if the reviser chooses L at $t = 0$. This means that h must be the best response to the following strategy:

$$L^* = s_R^{**} = (L, f(h, \tau) = H, f(l, \tau) = L). \tag{10}$$

Under L^*, the payoff to the committed player from choosing h conditional on a revision opportunity arriving at time τ is:

$$u_C(L^*, h; \tau) = \underbrace{x\tau}_{(L,h)} + \underbrace{z(1-\tau)}_{(H,h)}. \tag{11}$$

Intuitively, it is a sum of the cost of mis-coordination/conflict occuring before τ and the gain from winning occuring after τ. Taking the expected value one obtains:

$$E(u_C(L^*, h; \tau)) = x\mu + z(1-\mu). \tag{12}$$

In contrast, the payoff to the committed player from choosing l conditional on a revision opportunity arriving at time τ is $u_C(L^*, l; \tau) = w$ irrespective of τ. Therefore, if

$$x\mu + z(1-\mu) > w, \tag{13}$$

then the committed player will play h in any SPE, and hence the reviser will play H^*. The committed player will win the game. Rearranging terms in (13) one obtains (7). □

Intuitively, the condition states that the expected time to revision should be sufficiently short to justify the initial payoff loss from the preceding coordination failure. The reader can think of some initial investment of the committed player, and a subsequent reward for securing its preferred SPE.

The fact that all the relevant information about the revision timing are compressed into the mean of the probability distribution is an advantage of our framework that simplifies the exposition.

3.2. **Pure Coordination and Stag Hunt.** In both games the restriction on payoffs is identical to (5) with one difference, namely

$$a < d. \tag{14}$$

Therefore, for the committed player to win the game the necessary and sufficient threshold will be equivalent to that of the Battle of the Sexes game not only qualitatively but also quantitatively. Put differently, not only does (7) apply, but also the threshold $\bar{\mu}$ from (8) applies.

In terms of the Stag and Hunt game, in which

$$\begin{cases} a + b > c + d, \\ w + y > x + z, \end{cases} \tag{15}$$

this means that the Pareto-dominant rather than the risk-dominant outcome is selected. Such result is in contrast to the literature which will be discussed in Section 4.

3.3. Game of Chicken. The restriction on payoffs is:

$$c > d > b > a \quad \text{and} \quad x > z > y > w. \tag{16}$$

For illustration, let us use a specific (and symmetric) set of payoffs satisfying (16), namely

(17)

	l	h
L	$-10, -10$	$-1, 1$
H	$1, -1$	$0, 0$

The game has two pure strategy Nash that are asymmetric, and one mixed Nash. Therefore, the above discussion of equilibrium selection problems in regards to the Battle of the Sexes carries over as well, and so does the necessary and sufficient condition (7). The proof of Theorem 1 still applies here, only the analog of the necessary and sufficient condition (13) becomes

$$\underbrace{z\tau}_{(H,h)} + \underbrace{x(1-\tau)}_{(L,h)} > \underbrace{w}_{(L,l)}. \tag{18}$$

Rearranging yields the following expected revision threshold

$$\bar{\mu} = \frac{x-y}{x-w} \overset{(17)}{=} \frac{2}{11}.$$

3.4. Dynamic Inconsistency. The restriction on payoffs is:

$$a > c, d > b \quad \text{and} \quad y > z > w > x. \tag{19}$$

In the monetary policy context, see e.g. Barro and Gordon (1983), the column (committed) player can be interpreted as the policy maker and the row player (reviser) as the public. For illustration, let us use a specific (and symmetric) set of payoffs satisfying (19), namely

(20)

	l	h
L	$-1, -1$	$-2, -2$
H	$-2, 2$	$0, 0$

While the (H, h) outcome is Pareto efficient, the unique Nash equilibrium of the normal form game is the inefficient (L, l). This is due to the policy maker's temptation to surprise inflate (arising from a desire to boost output due to which $y > z$). The rational public however anticipates this and will not be surprised; for more details see Kydland and Prescott (1977).

For (H^*, h) to be the unique SPE, the necessary and sufficient condition (7) still has to hold. Intuitively, the revision time must be sufficiently short to compensate the policy maker for giving up the inflation surprise. The difference is that two conditions now have to be satisfied. Specifically, h must be the unique best response not only to L^* (as in all the above conditions) but also to H^*. This implies the following two conditions analogous to (13)

$$x\mu + z(1-\mu) > w \quad \text{and} \quad z > y\mu + w(1-\mu), \tag{21}$$

and, after rearranging

$$\mu < \frac{z-w}{z-x} \overset{(20)}{=} \frac{1}{2} \quad \text{and} \quad \mu < \frac{z-w}{y-w} \overset{(20)}{=} \frac{1}{3}. \tag{22}$$

Therefore, the threshold $\bar{\mu}$ in (7) will equal

$$\bar{\mu} = \min\left\{\frac{z-w}{z-x}, \frac{z-w}{y-w}\right\} \overset{(20)}{=} \min\left\{\frac{1}{2}, \frac{1}{3}\right\} = \frac{1}{3}. \tag{23}$$

4. Evolutionary (Stochastic) Stability

So far we have derived necessary and sufficient conditions that guarantee the dymanic revision game to have a unique pure strategy SPE in each of the considered game. However, even if these conditions are not satisfied the symmetry breaking by a positive revision probability allows us to apply standard evolutionary equilibrium selection techniques. Let us consider the Battle of the Sexes game and assume the following symmetric payoffs

(24)

	l	h
L	$z, 1$	$0, 0$
H	$0, 0$	$1, z$

where $z > 1$. The symmetry discussed in Section 3.1 prevents any anonymous dynamics to select between the two pure Nash equilibria in normal form game.

Applying standard evolutionary equilibrium selection techniques in the dynamic revision game is not trivial because of the complicated strategy space of the reviser. Therefore, we will first construct a 2×2 game that can reproduce all subgame perfect equilibrium payoffs of the dynamic revision game, namely:

(25)

	l	h
L^*	$z, 1$	$1-\mu, z(1-\mu)$
H^*	$z(1-\mu), 1-\mu$	$1, z$

The payoffs are obtained in the same way as (11). For technical reasons we will assume that

$$z(1-\mu) < 1. \tag{26}$$

For the game with no revision opportunities $\mu = 1$ and condition (26) holds.

Our next objective is to analyze the stochastic stability of pure strategy equilibria. The stochastic stability concept relates to a situation in which the players play the simultaneous move game many times with randomly selected opponents. Let proportion of the L^*-strategist in the population be p and proportion of the $l-$strategists be q. Then the expected payoff of the $l-$strategist is

$$u_C(l) = \underbrace{p}_{(L^*,l)} + \underbrace{(1-\mu)(1-p)}_{(H^*,l)}, \tag{27}$$

while the expected payoff of the $h-$strategist is

$$u_C(h) = \underbrace{z(1-\mu)p}_{(L^*,h)} + \underbrace{z(1-p)}_{(H^*,h)}. \tag{28}$$

It therefore follows that

$$u_C(l) - u_C(h) = p\mu(1+z) - (z+\mu-1), \tag{29}$$

and similarly

$$u_R(L^*) - u_R(H^*) = q(z + 2\mu - 1) - \mu. \tag{30}$$

It is easy to see that the game in (25) has three Nash equilibria (L^*, l), (H^*, h), and (p^*, q^*), where

$$p^* = \frac{z + \mu - 1}{\mu(1 + z)} \text{ and } q^* = \frac{\mu}{z + 2\mu - 1}. \tag{31}$$

Assume that the players revise their choices in the light of their experiences in such a way that the expected change in a fraction of population following a particular stratgy is governed by a payoff monotone dynamics. For concreteness, we will use the replicator dynamics[11]:

$$\begin{array}{c} E(dp) = p(1-p)(u_R(L^*) - u_R(H^*))dt \\ E(dq) = q(1-q)(u_C(l) - u_C(h))dt \end{array}. \tag{32}$$

Evolution of p and q is also subject to some noise. Following the literature we will call a Nash equilibrium stochastically stable if in the long-run the players find themselves almost all the time in its vicinity as the noise goes to zero.

To investigate stochastic stability of the equilibria of the game in (25) consider a system

$$\begin{array}{c} \frac{dp}{dt} = p(1-p)(q(z + 2\mu - 1) - \mu), \\ \frac{dq}{dt} = q(1-q)(p\mu(1+z) - (z + \mu - 1)). \end{array} \tag{33}$$

and note that system (33) has five steady states - two assymptotically stable and three unstable. In terms of the unstable ones, they are $(1, 0)$ and $(0, 1)$ and $\left(\frac{z+\mu-1}{\mu(1+z)}, \frac{\mu}{z+2\mu-1}\right)$, where the coordinates denote the equilibrium values of p and q. The former two correspond to monomorphic disequilibrium populations, while the latter corresponds to the mixed strategy Nash equilibrium.

In terms of the stable states, they are $(1, 1)$ and $(0, 0)$ which correspond to the Nash equilibria (L^*, l) and (H^*, h) respectively. The basins of attraction of the stable steady states are separated by a separatrix, which passes through the unstable steady states and satisfies the following equation in full differentials:

$$q(1-q)(p\mu(1+z) - (z + \mu - 1))dp - p(1-p)(q(z + 2\mu - 1) - \mu)dq = 0. \tag{34}$$

Define radius, R, of the equilibrium (H^*, h) as the minimal distance from it to the separatrix and its coradius, CR, as the minimal distance from (L^*, l) to the separatrix. Equilibrium (H^*, h) is stochastically stable if

$$R > CR, \tag{35}$$

(Ellison, 2000). Intuitively condition (35) implies that it takes more mutations to escape the basin of attraction of (H^*, h) than to get to it. We are going to show that for any $\mu < 1$ the separatrix passes above the line $p + q = 1$ and hence the condition (35) is

[11]Behaviorally, the replicator dynamics can be derived from an imitation and aspiration model in the case of a large population, see e.g. Samuelson (1997).

satisfied. Therefore, the equilibrium (H^*, h) corresponding to $p = q = 0$ is stochastically stable.[12] Formally, the following lemma holds:

Theorem 2. *Consider the Battle of the Sexes game described by (5). For any revision function $F(t)$ such that $\mu < 1$ the preferred equilibrium of the committed player (H^*, h) is stochastically stable.*

Proof. First, let us compute

$$\frac{d}{d\mu}(p^*(\mu) + q^*(\mu)) = -\frac{1}{\mu^2}\frac{(z-1)(z^2 - z(2+\mu^2-4\mu) + 3\mu^2 - 4\mu + 1)}{(z+1)(z+2\mu-1)^2} < 0 \tag{36}$$

for $\mu \leq 1$. The last inequality follows from the fact that the discriminant, D, of the quadratic polynomial

$$z^2 - z(2+\mu^2-4\mu) + 3\mu^2 - 4\mu + 1, \tag{37}$$

is given by:

$$D = (2+\mu^2-4\mu)^2 - 4(3\mu^2 - 4\mu + 1) = \mu^2(\mu^2 - 8\mu - 1) < 0 \tag{38}$$

for $\mu \leq 1$. This, together with the condition

$$p^*(1) + q^*(1) = 1 \tag{39}$$

implies that the mixed strategy Nash equilibrium lies above the line $p + q = 1$. See Figure 2 for a graphical demonstration. □

Let us note two things. First, the separatrix passes through the point $(0, 1)$. Second, equation (34) implies that for any $p \leq p^*$ if $p + q = 1$ then $p'(q) > -1$. Therefore by the single crossing property applied to the part of separatrix left of the mixed strategy, it will cross the line $p + q = 1$ only at point $(0, 1)$, and lie above this line everywhere else. Similar logic, applied to the part of the separatrix to the right of the mixed strategy Nash equilibrium implies that it strictly lies above the line $p + q = 1$ everywhere except points $(0, 1)$ and $(1, 0)$ where it crosses them.

5. Summary and Conclusions

Many real world situations related to economics and other disciplines are modelled using games that have inefficient and/or multiple Nash equilibria. If such models are to be used to draw predictions one has to address the issues of efficiency and equilibrium selection.

The literature has offered various ways to eliminate implausible equilibria containing incredible threats, implausible beliefs, and weekly dominated strategies. With the rise of evolutionary game theory it became for the first time possible to select among strict Nash equilibria in normal form games. The main weakness of the equilibrium selection approach is that the selected equilibrium in sensitive to the modelling decisions, for

[12] In contrast, in the case of $\mu = 1$ the reviser does not get a revision opportunity before the end of the game, and hence the dynamic revision game becomes equivalent to the normal form game. In this case the separatrix is given by equation $p + q = 1$ and pure strategy equilibria map into each other under the axial symmetry with respect to the separatrix. Therefore, neither of the equilibria is stochastically stable.

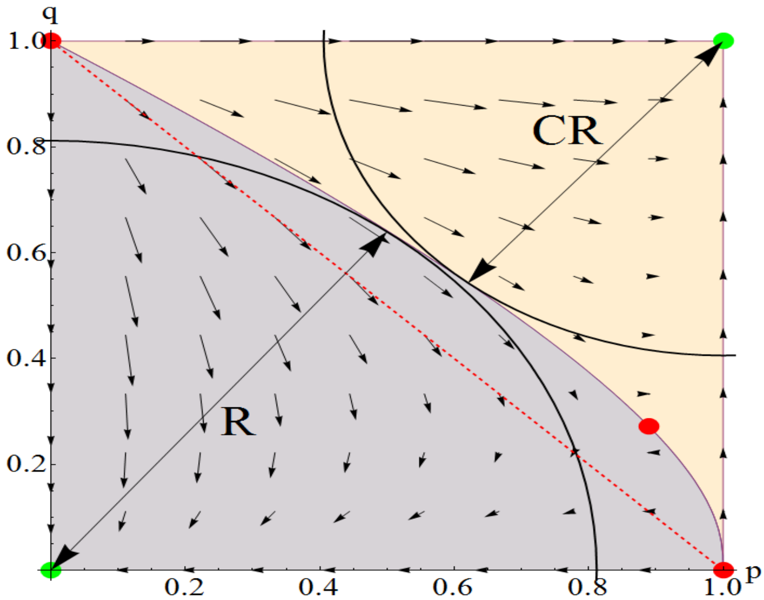

FIGURE 2. The basins of attraction and stochastically stable states.

example, to inclusion of strictly dominated strategies (see Kim and Wong (2009) who recently make this point in a general case).

Though the seemingly irrelevant details of the real life situation left out of the model may complicate the issue of equilibrium selection, they may also help to avoid the issue altogether. In this paper we show that the problems of inefficiency and equilibrium multiplicity may be avoided or reduced if one enriches the timing of the game. Allowing for a stochastic revision opportunity of one player we demonstrate that such symmetry breaking allows us to select a unique and efficient equilibrium under a broad set of conditions (whereby the requirements for subgame perfection are stronger than for evolutionary stability). This is also in games in which existing equilibrium selection techniques are powerless since the equilibria of the normal form game map into each other when one changes the names of the players (e.g. the Battle of the Sexes and the Game of Chicken).

A more general lesson of this paper is that often symmetry of the game leading to multiple equilibria may be a modelling artifact rather than a fundamental feature of reality. In that case rather than trying to solve the problems of inefficiency and multiplicity of equilibria it may be preferrable to go back to the modelling stage and enrich the game with some details that were abstracted at the first approximation: asynchronous timing, some differences in technological capabilities, idiosyncratic preferences, etc.

Let us point out that while we assume throughout that the committed player cannot revise his actions at all, the analysis suggests that the nature of the results carries over to a setting in which both players have a stochastic revision opportunity. This is because what matters is whether one player's revision opportunity is expected to arrive sufficiently late *relative* to the opponent's. We leave explicit modelling of such a setting for future research.

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