Monetary and Fiscal Policy Interaction With Various Degrees of Commitment[1]

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Abstract

Prior to the global financial crisis, the stance of fiscal policy in a number of countries had raised concerns about risks for the outcomes of monetary policy. In order to assess these concerns we examine the fiscal-monetary interactions in an asynchronous game theory framework which generalizes the standard commitment concept by making it dynamic. The framework allows for concurrent commitment and partial commitment, ie both policies may be committed at the same time, and may be committed for different periods of time. It is demonstrated that undesirable outcomes can be prevented if long-term monetary commitment is sufficiently strong relative to fiscal rigidity and ambition. Moreover, the threshold for monetary commitment - that is a function of various structural and policy parameters - can not only resist fiscal pressure, but also discipline an ambitious government by counter-acting their excessive policies. This regime can achieve socially optimal medium/long-run outcomes for both policies. The policy implications are obvious: by committing more explicitly to long-term price stability (eg a numerical target for average inflation), the Fed, the ECB, and others would not only ensure their credibility, but also indirectly induce a reduction in the average size of the budget deficits and debt in their economies.

Keywords: commitment; monetary vs fiscal policy interaction; extensive games; asynchronous moves; Battle of the sexes; explicit inflation targeting;
JEL classification: E63, C73

[1]We are grateful to Don Brash, the Governor of the Reserve Bank of New Zealand during 1988-2002, for valuable views and for contributing the case study of Section 8. We would also like to thank Iris Claus, Damien Eldridge, Frank Hespeler, Jeff Sheen, Lawrence Uren, and the participants of the 24th Australasian Economic Theory Workshop, 12th Australasian Macroeconomics Workshop, the Central European Program in Economic Theory workshop, and CAMA's 2007 Fiscal Policy Frameworks Conference for useful suggestions and comments and the Australian Research Council for financial support (DP0879638). All errors and omissions are our own responsibility.

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1. Introduction

Consider the following situation. A fiscally prudent political party makes the claim that, as a result of their fiscal restraint, inflation would be significantly higher under a 'less-prudent' rival party. The less-prudent party argues the claim is misplaced since the country has an independent and responsible central bank. Which party was right? And under what circumstances?

This is far from a hypothetical scenario, and it highlights the importance of understanding the interaction of fiscal (F) and monetary (M) policy on outcomes of *both* policies. Let us stress from the outset that our interest lies in *long-run outcomes* of the interaction - the policy responses to the financial crisis of 2007-8 are therefore orthogonal to our analysis and results.

The idea that M and F policies might interact goes back to Tinbergen (1954) and Mundell (1962), but until recently most models used for policy design treated each policy in isolation. The subsequent literature has mainly examined *direct* institutional interventions - the ability of the government (F policymaker) to affect M policy outcomes through the appointment of the central banker (Rogoff (1985)), optimal contract with the central banker (Walsh (1995)), or through overriding the central banker (Lohmann (1992)).[5]

The focus of this paper is *indirect interaction* which is more subtle and less well understood.[6] It works through spillovers of economic outcomes – variables such as inflation, output, debt, exchange rate, asset prices, or consumer confidence are all affected by *both* policies. And they in turn affect the optimal setting of *both* policies. Most obviously, an unwarranted increase in government spending commonly leads to higher output and possible deviations from the central bank's preferred inflation level.[7]

Our contribution in this paper is to examine the policy interaction in a dynamic game theoretic setting, in which policymakers may have different degrees of commitment to their particular regimes/policies, and to show that some conventional results may be refined or even qualified. Specifically, our innovation is to develop an asynchronous game framework that generalizes the alternating move games of Maskin and Tirole (1988) and Lagunoff and Matsui (1997).[8]

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[5] For an alternative analysis of this direct interaction, which encompasses all three approaches and formally connects them to the reputation literature initiated by Backus and Driffill (1985), see Hughes Hallett and Libich (2007).

[6] See eg Adam and Billi (2008), Eusepi and Preston (2008), Chadha and Nolan (2007), Persson et al. (2006), Benhabib and Eusepi (2005), Gali and Monacelli (2005), Hughes Hallett and Weymark (2005), Eggertsson and Woodford (2004), Dixit and Lambertini (2003) and (2001), van Aarle, et al. (2002), Leeper (1991) or Sargent and Wallace (1981).

[7] The classic reference on the importance of the interactions between domestic policies is Cooper (1969). A more modern discussion, triggered by the policy dilemmas of the current recession, will be found in Cochrane (2009) who shows how all fiscal policies have monetary consequences, and vice versa, such that in the limit they merge to become indistinguishable.

[8] The existing game theoretic work provides a strong motivation for our general approach. For example, Cho and Matsui (2005) argue that: *'[a]lthough the alternating move games capture the essence of asynchronous decision making, we need to investigate a more general form of such processes. . . '*. However, that generalisation has never been made. The contribution of this paper is to provide that generalization, and to show the benefits and implications of an asynchronous game between fiscal and monetary policies.

This framework features a combination of simultaneous and sequential moves, and allows actions to differ in frequency as suggested by Tobin (1982). That enables us to postulate a new game theoretic concept of commitment that has several advantages over the standard concept of Stackelberg leadership - it is more general and its dynamic features make it more realistic in many contexts. Most importantly, without compromising its tractability, it allows for: (i) concurrent commitment of more than one player/policy, (ii) partial commitment, and (iii) an endogenously determined (optimally selected) degree of commitment.

Setup. We are interested in macroeconomic settings in which: (1) M and F policies are formally independent, but still inter-related (affect each other) through spillovers on the target(s) of the other policy, and (2) there exist some inefficiency or time-inconsistency, coming from political economy factors on the F policy side. Specifically, the F policymaker (government) will be excessively *ambitious* in its output objectives, whereas the M policymaker (the central bank) will be *responsible* in this respect.[9]

In order to best illustrate the game theoretic insights, and keep the analysis general, we do not use a specific macroeconomic model (a formal analysis based on a fully articulated macroeconomic model is available: see Libich, Hughes Hallett, and Stehlík (2007); henceforth LHHS). Instead, we discuss how any analytically tractable model of policy interaction can be mapped into a simple game theoretic representation, and analyzed using familiar techniques of non-cooperative game theory.

Consider two medium/long-run options for each policy: 'discipline', D, ie delivering the socially optimal levels on average, and 'indiscipline', I, ie delivering the discretionary levels that however are, due to F ambition, socially inferior. In terms of F policy D and I can be interpreted as running, *on average*, a balanced budget vs a deficit. In terms of M policy, D and I can be interpreted as average low and high (debt-monetizing) inflation.[10]

Following Sargent and Wallace (1981), our main attention lies in M-F interactions that has the form of a *coordination game*, and specifically *the Battle of the sexes.* Figure 1 gives an example of this game, which has two pure strategy Nash equilibria, (MD, FD) and (MI, FI), and one in mixed strategies.

There are three reasons for this choice. First, it is the most interesting scenario from the game theoretic point of view as there are equilibrium selection problems - into which our framework provides some novel insights. Specifically, the game features both a *coordination problem* (to avoid the mixed Nash equilibrium) and a *conflict* (to secure the preferred pure Nash).

Second, these two features are present in a wide range of policy interaction models, eg Adam and Billi (2008), Branch, et al. (2008), Resende and Rebei (2008), Hughes Hallett and Libich (2007), Dixit and Lambertini (2003) and (2001), Blake and Weale (1998),

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[9] In a reduced-form model, a responsible policymaker can be thought of a targeting potential output, whereas an ambitious policymaker's output target is above potential.

[10] Let us stress again that as our focus is on the long-run macroeconomic outcomes. In this focus we follow Sargent and Wallace (1981), Alesina and Tabellini (1987), Nordhaus (1994) and the subsequent literature. This implies that D and I should be interpreted as setting *average* levels. Furthermore, we do not restrict our attention to a particular measure of inflation such as the consumer price index - the term inflation should throughout be interpreted broadly as potentially including various specifications.

		F	
		D	I
M	D	1, 0	-½, -½
	I	-1, -1	0, 1

FIGURE 1. The Battle of the Sexes scenario: an example

Nordhaus (1994), Sims (1994), Woodford (1994), Leeper (1991), Wyplosz (1991), Petit (1989), or the seminal work of Sargent and Wallace (1981).[11]

Third, the results derived in the Battle of the sexes will imply analogous results for alternative scenarios (including the Game of Chicken) - they will only differ quantitatively, not qualitatively.

Standard (Static) Commitment: Only One Player and One Degree. The standard game theoretic concept of commitment involves Stackelberg leadership, ie the first move. This means that only one player, the leader, can be committed at a time (the other one is the follower). Further, it is impossible to study a partial or incomplete degree of commitment.

Introducing this standard commitment in the Battle of the sexes uniquely selects one of the pure Nash equilibria, one where having the first move (leadership) is an advantage. Under F commitment - a situation often called F dominance following Sargent and Wallace (1981) - the F's preferred and socially undesirable outcome (MI, FI) results. This can also be interpreted as the case of active F and passive M policy case examined by Leeper (1991). Conversely, under M commitment (dominance), the M's preferred and socially optimal outcome (MD, FD) obtains. Our contribution lies in broadening and refining these conventional results by allowing for various degrees of commitment.

Generalized (Dynamic) Commitment: Any Number of Players and any Degree. We introduce the idea of asynchronous games as a way to overcome the restrictions of the standard simultaneously repeated game. In this paper we focus on *discrete* time with *deterministic timing* of moves in which the M-F interaction has been most often studied.[12]

Let us define, in the spirit of Taylor (1979), player i's **deterministic commitment** or **rigidity**, $r^i \in \mathbb{N}$, as *'the number of periods for which player i's action cannot be altered'*. This implies that, after the initial simultaneous move, every player i moves with some *constant frequency* - every r^i periods, see Figure 2 for an example

[11]The M-F interactions have often been modelled as the *Game of Chicken*, eg Barnett (2001), Bhattacharya and Haslag (1999), Artis and Winkler (1998), and Alesina and Tabellini (1987). The Battle of the Sexes is similar in that it also features two pure Nash equilibria, (MD, FI) and (MI, FD), each preferred by a different player, and one Nash in mixed strategies. Nevertheless, LHHS (2007) show that an *anti-coordination* game such as the Game of chicken is unlikely to arise under a responsible central bank, since it has no *structural* temptation to inflate if the government is disciplined over the long-term.

[12]A related paper Libich and Stehlík (2009) examines continuous time and stochastic timing, with a focus on a monetary union.

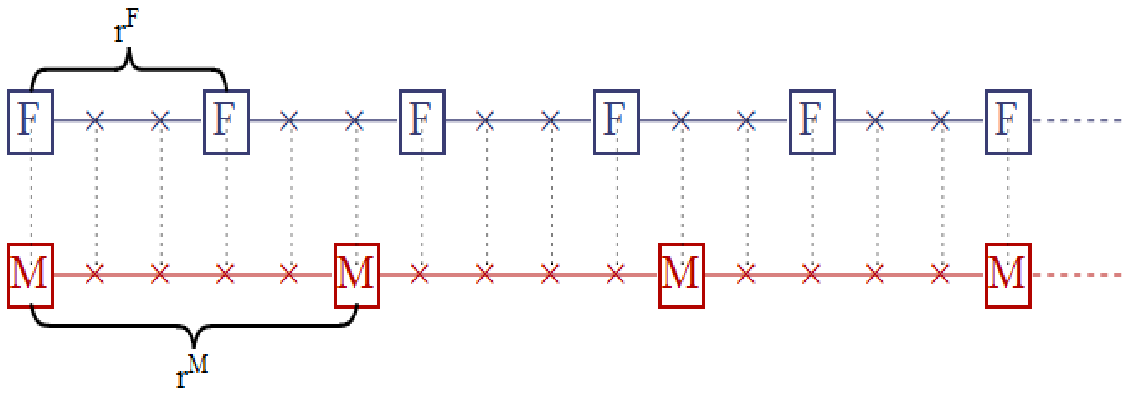

FIGURE 2. An asynchronous game with deterministic commitment/rigidity - an example of timing of moves with $r^F = 3$ and $r^M = 5$.

Let us note two important points. First, the concepts of commitment and rigidity are formally identical in our framework, both referring to the players' inability to move. Nevertheless, in the real world such inability comes from different sources, which we will acknowledge by referring to r^M as M commitment and r^F as F rigidity. Second, since the policy instruments are average levels, r^M and r^F express *long-run* (not short-run) commitment or rigidity.[13]

To compare the results to the standard repeated game we adopt all its main assumptions: the game starts with a simultaneous move, and all past actions are observable (ie games of 'perfect monitoring'). Our game thus combines perfect and imperfect information, which is arguably a good description of the real world M-F interaction. This deterministic framework further captures the observation of Tobin (1982) that '*Some decisions by economic agents are reconsidered daily or hourly, while others are reviewed at intervals of a year or longer...*', and follows Tobin's call: ...'*It would be desirable in principle to allow for differences among variables in frequencies of change and even to make these frequencies endogenous...*'.

A Preview of the Findings. We are interested in circumstances under which there is unique equilibrium selection in the Battle of the sexes. In particular we derive cirumstances under which one player *surely-wins*, under which his preferred outcome *uniquely* obtains on the equilibrium path of all subgame perfect Nash equilibria (SPNE) of the asynchronous stage game. The analysis offers four main findings:

(1) In order for player i to surely-win the Battle of the sexes, his *relative* commitment/rigidity vis-à-vis the opponent's has to be sufficiently strong. Formally, it must hold that $r^i > \overline{r^i}(r^j, .) \geq r^j$, where the threshold is not only an increasing function of the opponent j's commitment, but also dependent on other variables

[13] Therefore, r^M should *not* be interpreted as the frequency of the central bank's interest rate decisions. Instead, r^M describes how difficult it is to reconsider the inflation target for average inflation. Section 3 argues this to be an increasing function of how *explicitly* long-term price stability is grounded in the central banking statues and legislation.

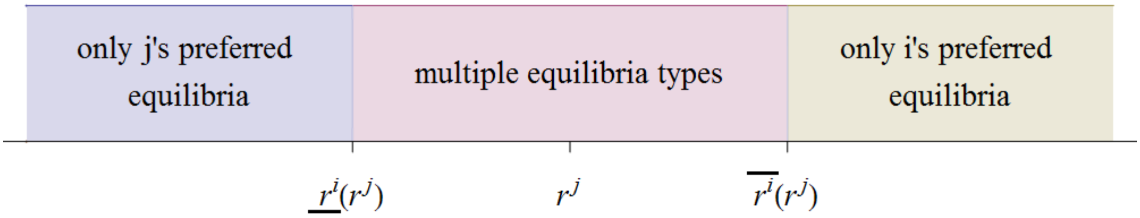

FIGURE 3. The r^i space featuring the thresholds and regions of equilibria.

such as the players' discount factors and the structure of the economy. This implies that if $r^i < \underline{r^i}(r^j) \le r^j$ (where $\underline{r^i}(r^j)$ is some 'mirror-image' lower bound of $\overline{r^j}(r^i)$) then player i *surely-loses*. That is to say no SPNE exists with the player's preferred outcome throughout the equilibrium path. Figure 3 summarizes these results graphically, showing the thresholds as well as the region of multiple types of equilibria between them.

(2) Interestingly, under some circumstances, the required degree of relative commitment/rigidity is arbitrarily low, $\overline{r^i}(r^j) = r^j$. In Figure 3 this would mean, if satisfied for both players, that the region of multiple types of equilibria becomes a singleton with $r^i = r^j$.

(3) In contrast, under other circumstances, namely a very impatient policymaker, even an infinitely strong commitment, $r^i \to \infty$, is insufficient to surely-win. In other words, the $\overline{r^i}$ threshold does not exist, which would mean that in Figure 3 we only have two regions or one region (if one or both players, respectively, are sufficiently impatient).

(4) Furthermore, and perhaps most surprisingly, under some circumstances of the multiple equilibria region, $r^i \in \left(\underline{r^i}, \overline{r^i}\right)$, even if player i is more strongly committed than player j, $r^i > r^j$, player i *likely-loses* and player j *likely-wins*. In other words, i's preferred outcome cannot be obtained in equilibrium, whereas j's can.

Further, one of the paper's contributions is to contrast these results with those obtained under the standard static concept of commitment, and show that they differ substantially. This is because, under static commitment, the committed player (leader) *always* ensures its preferred outcome in the Battle of the sexes is achieved - unlike findings (3)-(4). Furthermore, this happens irrespective of any structural model or choice of policy variables - unlike findings (1)-(3).

Policy Implications. These findings are mixed news for the outcomes of the policy interactions. The bad news is that undesirable M policy outcomes may obtain even if the central bank is independent, responsible, patient, and (somewhat) committed. As Sargent and Wallace (1981) put it: *'...Friedman's list of the things that monetary policy cannot permanently control may have to be expanded to include inflation.'* In other words, in the presence of fiscal policy these central bank characteristics are not sufficient on their own for long-term price stability or M policy credibility.

The good news is that a solution to the commitment problem still exists. Our analysis shows that if a responsible central bank commits *sufficiently strongly* relative to F

policy's rigidity and ambition, then it can safeguard its credibility and the preferred low-inflation level. Furthermore and as a by-product, this can indirectly discipline the F policymaker and achieve the socially optimal disciplined outcomes for *both* policies throughout the medium/long-run. Intuitively, if long-term price stability is sufficiently explicitly stated in the statutes/legislation with the central banker being accountable for achieving it, this creates incentives for the government to balance its budgets on average since there is no chance of M policy accommodating a deficit, and hence little or no boost to output. We are able, moreover, to identify the thresholds that define what constitutes 'sufficient' in this context.

The implication for M policymakers in countries with ambitious F policymakers is the following. In order to discourage and/or counter-act an excessive F stance, they should if possible make their commitment to long-term price stability (a low-inflation target) more explicit. The recommendation for F policymakers trapped with unsustainable F plans is that *imposing* M commitment onto the central bank (legislating a long-run numerical inflation target) may help the government later justify or gain political support for a necessary F reform and consolidation.

Empirical Evidence. The paper has several implications that can be taken to the data. In particular, for *some but not all* parameter values, a more explicit long-term M commitment: (i) can reduce the level and the variability of inflation; (ii) can act as a substitute for central bank *goal*-independence in achieving credibility; and (iii) can discipline F policy and induce reductions in the average size of the budget deficit and debt.

LHHS (2007) examine these hypotheses using the existing literature and find fair support for these findings. The main difficulty is the quantification of the strength (explicitness) of M commitment. While there exist various indices in the literature measuring central bank accountability and goal-transparency, they have not generated robust results and have not been generally accepted. For that reason their use as a proxy for the strength of M commitment remains controversial. For that reason we do not include a detailed empirical evaluation in this paper; we merely report several relevant papers.

Instead, Section 8 presents a case study written by Dr Don Brash, Governor of the Reserve Bank of New Zealand during 1988-2002, in relation to the above finding that M policy can discipline F policy. His contribution describes the developments in New Zealand shortly after the (world's first) adoption of an *explicit* commitment to a low-inflation target, and highlights the 'disciplining' effect of this M policy arrangement on F policy.

The rest of the paper proceeds as follows. Section 2 describes the macroeconomic features of the policy interaction. Section 3 introduces our generalized dynamic commitment and Sections 4-6 then use it to reinvestigate conventional conclusions made under static commitment. Section 7 extends the analysis to a monetary union with heterogeneous F policies. Section 8 presents a case study of New Zealand. Section 9 summarizes and concludes.

2. The Policy Interaction as the Battle of the sexes

The macroeconomic setting we are interested in has the following properties - present in most of the papers cited above: 1. M and F policy are independent in a legal and political sense, but they are still inter-dependent since their outcomes spill over to the other policy. This is because the policies share some objectives, and can both affect these shared objectives. 2. Some of the long-term outcomes will deviate from the social optimum due to political economy or structural factors on the F side. Specifically, one can think of an attempt to get re-elected (in the presence of naïve voters, lobby groups, unions etc), or inherited and persistent F settings that automatically tip the budget into a deficit (eg unaffordable welfare/health/pension schemes, or high debt).

For clarity and the reasons given in the introduction, we continue with the 2x2 specification of the Battle of the sexes game. It is important to realize that this type of interaction featuring both a coordination problem and a policy conflict arises in *some* macroeconomic models under *some* parameter values - alternative cases will be discussed below.[14] In addition to the *specific* Battle of the sexes game as reported in Figure 1, namely

$$a = z = 1 > d = v = 0 > b = w = -0.5 > c = y = -1, \tag{1}$$

we will also depict *general* payoffs to better show the various influences. Specifically, the game is summarized in Figure 4, where the $\{a, b, c, d, v, w, y, z\}$ payoffs satisfy the following constraints

$$a > c, a > d > b \text{ and } z > y, z > v > w. \tag{2}$$

		F	
		FD	FI
M	MD	a, v	b, w
	MI	c, y	d, z

Figure 4. General payoffs

Recall that there are two pure strategy Nash equilibria, (MD, FD) and (MI, FI), whereby the former is preferred by M and assumed socially optimal, whereas the latter is preferred by F and is socially inferior. In addition, there is a mixed strategy Nash equilibrium which yields lower payoffs to both policymakers than either of the pure Nash equilibria.[15]

[14] Both features are present in, among others, Adam and Billi (2008), Branch, et al. (2008), Resende and Rebei (2008), Hughes Hallett and Libich (2007), Benhabib and Eusepi (2005), Dixit and Lambertini (2003) and (2001), Blake and Weale (1998), Nordhaus (1994), Sims (1994), Woodford (1994), Leeper (1991), Wyplosz (1991), Petit (1989), Alesina and Tabellini (1987), or Sargent and Wallace (1981).

[15] It is straighforward to show that the expected payoff from the mixed Nash of the specific game in (1) is -0.2 to both players.

The above discussion implies the payoffs to be some functions of the structural and policy parameters of the underlying macroeconomic model. Since the Battle of the sexes setup can be produced via fundamentally different models (see the references cited above), one obviously cannot write down a general mapping between the deep parameters and payoffs. Nevertheless, it can be done once a specific macroeconomic model has been selected.

LHHS (2007) offer an example of this following the Cho and Matsui (2005) approach. Using a reduced-form model like that of Nordhaus (1994) with a standard quadratic utility function for both policymakers (but a higher output target of F), the analysis implies that we can interpret the value of $(a - b)$ as the per-period *output cost* of a policy conflict (including disinflation costs), whereas $(a - d)$ expresses the *inflation cost* of M indiscipline.

The justification for why even a benevolent and responsible central bank may choose debt monetization and deviate from the socially optimal level may differ across the macroeconomic settings with each model potentially offering a slightly different explanation.

For example, Sargent and Wallace's (1981) unpleasant monetary arithmetic requires the central bank to generate seigniorage revenues to prevent the government's default. A parallel avenue is Leeper (1991) and the Fiscal theory of the price level literature, where M policy is forced to be passive by an active F policy resulting in permanent changes in M responses to F shocks. As an alternative explanation, if there are frictions in the economy and the policy instruments are substitutes in affecting output, they may be used according to comparative advantage to minimize the various distortions which might lead to an inferior MI type equilibrium (see eg Adam and Billi (2008) or Resende and Rebei (2008)).

Conversely, the reason for even ambitious governments to choose to be disciplined, FD, is the incentives implied by M policy responses. If the government knows that the central banker would fully counter-act its expansionary actions by strongly tightening the economy, and hence eliminating any desired output gains, the government's temptation for excessive F actions fade away.

The fact that the two pure strategy Nash equilibria are each preferred by a different player means that neither is more likely to be selected since the focal point argument cannot be used. A common solution to this problem is impose commitment. As noted in the introduction, the standard static commitment concept that imposes *Stackelberg leadership* on one player turns out to be an advantage. Under M's commitment (leadership/dominance), player M's preferred outcome (MD, FD) obtains in equilibrium, whereas under F commitment F's preferred outcome (MI, FI) results.

3. Generalized (Deterministic) Commitment

This section uses an asynchronous game framework to allow for (i) concurrent commitment of more than one player/policy, and (ii) their partial commitment. Our goal is to examine how the macroeconomic outcomes of the policy interaction vary with *various degrees* of M commitment and F rigidity.

3.1. Real World Interpretation of r^M and r^F.

It is important that the concepts of M commitment and F rigidity, r^M and r^F, are interpreted correctly. They were both defined above as the number of periods for which a player's freedom of action cannot be altered. Arguably, the inability to change the long-run policy course at every point in time is due to the fact that the regime and its objectives or future plans and strategies may be *legislated*.

Therefore, both r^M and r^F can be interpreted as the degree of *explicitness* with which the long-run targets, plans, or strategies of the respective policies are stated in the legislation/statutes. The underlying assumption is that the more explicitly a certain long-term goal/setting is grounded in legislation, the less frequently it can be altered (in the Taylor (1979) deterministic sense), or the less likely it is to be altered (in the Calvo (1983) probabilistic sense).

It is however essential to realize a key difference between M commitment and F rigidity, coming from the fact that the considered central banker is responsible, whereas the government is ambitious. In particular, the M policy objectives/targets, to which M commitment relates, are sustainable over the long-term whereas the targets or objectives of F policy may not be.[16]

Specifically, we can think of r^F as the degree with which various long-term fiscal schemes and settings that lead to excessive spending such as unsustainable welfare/health/pension legislation, or high debt repayments, are grounded in the related legislation. In addition, it is also affected by the political culture of the country, and the historical developments that determine the expectations of voters, and hence politicians.

In terms of r^M, the above discussion implies that M commitment is *not* one to specific actions (as in Taylor (1993) or the Barro-Gordon (1983) literature), nor the timeless perspective type of commitment of Woodford (1999), or the quasi commitment of Schaumburg and Tambalotti (2007). This is because the central bank is still able to choose the desired long-run policy level, or policy framework, every r^M periods, and the underlying short-run actions within that framework every period. There are no restrictions on *how* choices need to be made in any period. Instead, the central bank is pre-committed to the parameters of its particular *regime* since r^M cannot be altered throughout the game.[17]

A real world example of such M commitment is a numerical target for average inflation adopted by a number of countries in the past two decades. Specifically, as a demonstration of the deterministic nature of r^M, the 1989 Reserve Bank of New Zealand Act states that the long-run inflation target may only be changed in a Policy Target Agreement between the Minister of Finance and the Governor, and that this can only be done on *pre-specified regular* occasions (eg when a new Governor is appointed).[18]

3.2. **Assumptions.** Our framework adopts all the assumptions of a standard repeated game. First, the degrees of commitment/rigidity r^i are constant throughout each game.

[16] In Section 9 we will discuss this case of a responsible government.

[17] This interpretation is in line with Geraats' (2002) concept of 'political transparency'. Her concept has three elements, namely 'formal objectives', 'quantitative targets', and 'institutional arrangements', all of which are officially grounded in the policy's legal framework.

[18] Since late 1990 the PTA was 'renegotiated' five times, ie roughly every three years. On only two occasions was the target level changed: in 1996 from 0-2% to 0-3% and in 2002 to 1-3%. It should further be noted that the absence of a legislated numerical target may not necessarily imply $r^M = 1$; it has been argued that many countries pursue an inflation target implicitly (including the US, Goodfriend (2003); or the Bundesbank in the 1980-90s, see Bernanke, et al. (1999)). In such cases we have a value $r^M > 1$, but it is nonetheless lower than under an explicit target.

Second, they are common knowledge. Third, all past periods' moves can be observed. Fourth, the game starts with a simultaneous move. Fifth, players are rational, have common knowledge of rationality, and for expositional clarity they have complete information about the structure of the game and the opponent's payoffs.

In addition to our definition of deterministic commitment r^i in Section 1, we now have the following.

Definition 1. *An* ***asynchronous game with deterministic commitment/rigidity*** *is an extensive game that starts with a simultaneous move in period 1, and continues with moves every discrete* r^i *periods. Its* ***stage game*** *finishes after* T *periods, where* $T \in \mathbb{N}$ *denotes the 'least common multiple' of* $r^i, \forall i$.

An example of the stage game is presented in Figure 2 in the form of a time line depicting a case with $T(r^M = 5, r^F = 3) = 15$. Note that the stage game is now a dynamic (extensive-form) game, unlike in the simultaneously repeated game in which the stage game is static (normal-form).

3.3. **Notation.** Denoting n^i to be the i's player's n'th move (not period), and N^i the number of moves in the asynchronous stage game, it follows that $N^i = \frac{T(r^M, r^F)}{r^i}$. Also, M_n^l and F_n^l will denote a certain action $l \in \{D, I\}$ at a certain node n^i; eg F_2^I refers to F's indiscipline in its second move.

For the rest of this section we assume, without loss of generality, that $r^i > r^j$, where $i \in \{M, F\} \ni j$. We will denote $\frac{r^i}{r^j} \geq 1$ to be the players' *relative commitment/rigidity* . Also, $\left\lfloor \frac{r^i}{r^j} \right\rfloor \in \mathbb{N}$ will be the integer value of relative commitment (the *floor*) and

$$R = \frac{r^i}{r^j} - \left\lfloor \frac{r^i}{r^j} \right\rfloor = [0, 1)$$

denotes the fractional value of relative commitment (the *remainder*).[19]

Further, we denote $b(.)$ to be the best response. For example, $F_1^D \in b(M_1^D)$ expresses that F^D is F's best response to M's initial D move, and $b(M_1^D) = \{F_1^D\}$ expresses that it is the unique best response. Thus $F_1^* \in b(M_1)$ expresses that F's optimal/equilibrium play in move 1 is the best response to M's first move.

Finally, various threshold levels under which a player surely-wins (surely-loses) will be denoted by upper (lower) bar. More specifically, $\overline{r^i}$ will be some *sufficient* commitment level of player i (that obtains for all R), whereas $\overline{r^i}(R)$ will be i's *necessary and sufficient* commitment level that is a function of R.

3.4. **Recursive Scheme.** Throughout the proofs we will be taking advantage of the recursive scheme implied by the setup. Still assuming $r^i > r^j$, let k_n be the number of periods between the n^i-th move of player i and the immediately following move of player j (for an example, see Figure 5)

[19] It will be evident that R plays an important role as since it determines (in a non-monotonic way) the exact type of asynchrony in the game. Note that if $R > 0$ the players change in their leadership role throughout the stage game.

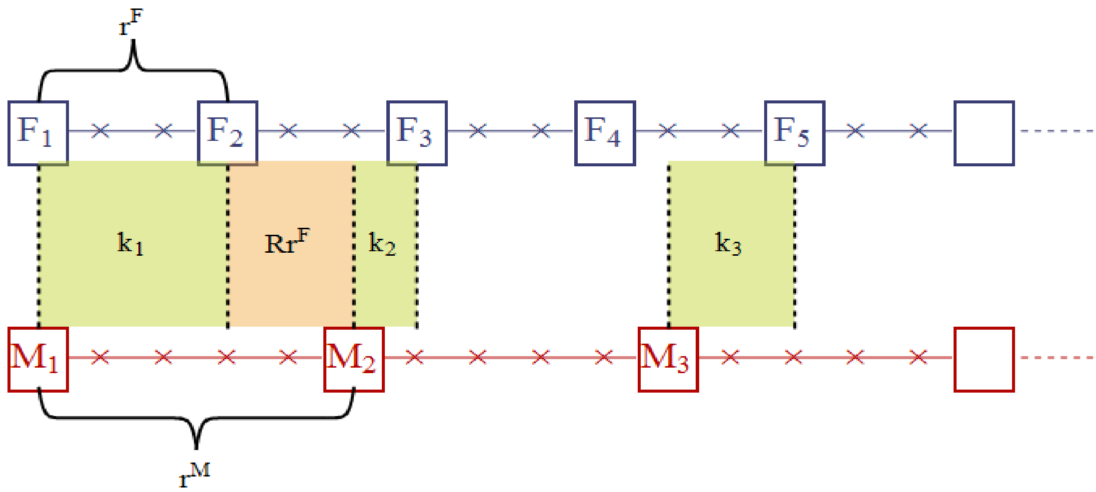

Figure 5. An asynchronous stage game with deterministic commitment: illustration of the recursive scheme and of R, k and n^i.

Using this notation we can summarize the recursive scheme of the game as follows:

$$k_{n+1} = \begin{cases} k_n - Rr^j & \text{if} \quad k_n \geq Rr^j, \\ k_n + (1-R)r^j & \text{if} \quad k_n < Rr^j, \end{cases} \tag{3}$$

Generally, k_n is a non-monotonic sequence.

3.5. **History, Future, Strategies, and Equilibria.** By convention, history in period t, h_t, is the sequence of actions selected prior to period t. And the future in period t is the sequence of current and future actions. It follows from our perfect monitoring assumption that h_t is common knowledge at t. Let us refer, following Aumann and Sorin (1989), to moves in which a certain action $l \in \{D, I\}$ is selected for all possible histories as *history-independent*.

A strategy of player i is a vector that, $\forall h_t$, specifies the player's play $\forall n^i$. The asynchronous game will commonly have multiple Nash equilibria. To select among these we will use a standard equilibrium refinement, subgame perfection, that eliminates non-credible threats. Subgame perfect Nash equilibrium (SPNE) is a strategy vector (one strategy for each player) that forms a Nash equilibrium after any history h_t.

Given the large number of nodes in the game we focus on the *equilibrium path* of the stage game SPNE, ie the actions that actually get played. In doing so we will use the following terminology regarding two symmetric types of SPNE we are interested in.

Definition 2. *Any SPNE of the asynchronous stage game of Definition 1 that has, on its equilibrium path, both policymakers playing: (i) D in all their moves, $i_n^{D*}, \forall n, i$, and (ii) I in all their moves, $i_n^{I*}, \forall n, i$, will be called* ***Disciplined*** *and* ***Indisciplined*** *SPNE respectively. Further, player i will be called to: (iii)* ***surely-win*** *the Battle (and the* *opponent j to **surely-lose**) if all SPNE of the game are of i's preferred type; and (iv) **likely-win** (and j **likely-lose**) if there exists at least one SPNE of i's preferred type, and there exists no j's preferred SPNE type.*

3.6. **(Non)-Repetition.** While the asynchronous game of Definition 1 can be repeated we will restrict our attention to its dynamic stage game. This is possible without loss of generality because we will be deriving conditions under which an *efficient* outcome *uniquely* obtains on the equilibrium path of the asynchronous stage game. Due to these two properties, if the derived conditions are satisfied, then repeating the game would not affect the reported equilibrium.[20]

Intuitively, if the *effective minimax values* of the players in the stage game (that are the infima of the players' subgame perfect equilibrium payoffs, see Wen (1994)) are unique and Pareto-efficient, then the effective minimax values of the repeated game (with any finite or infinite number of repetitions) will also be the same. Put differently, the set of Pareto-superior payoffs is empty as we are already on the Pareto-frontier. The uniqueness property also implies that we can only focus on pure strategies without loss of generality.

4. Results: Patient Policymakers

The next two sections focus on deriving the conditions under which M surely-wins (and F surely-loses) the Battle of the sexes, since this generates the socially optimal outcome. For illustration purposes we first examine the game under the assumption of patient policymakers and then consider the effect of their impatience (discounting). Furthermore, we support the results of the *general* game, where only (2) is required to hold, with those of the *specific* game in which (1) applies.

Proposition 1. *Consider the Battle of the sexes policy interaction in which* (2) *holds and* $\delta_F = \delta_M = 1$. *The sufficient condition,* $\forall R$, *for* M *to surely-win and* F *surely-lose is*

$$r^M > \overline{r^M} = r^F \left(\frac{a-b}{a-d} + \frac{v-w}{z-y+v-w} \right) > r^F. \tag{4}$$

In the specific game in which (1) *holds this reduces to*

$$r^M > \overline{r^M} = 1.7 r^F. \tag{5}$$

Proof. To prove that any SPNE will be Disciplined it suffices to show that, under the stated circumstances, I is never played on the equilibrium path. This can be done by showing that D is M's unique best play in *all nodes* n^M *for all histories* h_t. Put differently, every optimal move M_n is history-independent, ie not only $b(F^D) = \{M_n^D\}, \forall n$ (which is trivially satisfied), but also $b(F^I) = \{M_n^D\}, \forall n$. Intuitively, the latter ensures that M will find it optimal to 'fight' excessive F policies even if he knows with certainty that it will be costly and lead to more volatile output. As F's unique best response to MD is FD, this will then ensure FD throughout the equilibrium path as well. In words, upon realizing this determination of the central bank the government will find it optimal not to go ahead with the fight, and play discipline from the beginning.

[20] In this sense we can think of our analysis as the worst case scenario in which reputation cannot help in cooperation.

We solve the game backwards and prove that statement by mathematical induction with respect to M's moves, restricting our attention to the relevant region $r^M > r^F$. First, we prove that on the equilibrium path MD will be played in M's last move $n^M = N^M$ (the inductive basis). Specifically, part A) of the proof will examine the case $R = 0$, and part B) $R > 0$. Second, supposing that it holds for some $n^M \leq N^M$, we show in part C) that the same is true for $n^M - 1$ as well.[21] We will only show part A) in the main text to develop the intuition, and relegate the rest to the Appendix.

A) $\mathbf{n^M = N^M}$ **under** $\mathbf{R = 0}$. Here we have $T(r^M, r^F) = r^M$, and therefore $N^M = 1$ and $N^F = r^M$. Solving backwards, we know F would like to play the best response to M's actions, $F_n^* \in b(M_1), \forall n^F$. From her second move till the end of the dynamic stage game F can observe M_1, and will hence rationally play FD to M_1^D, and FI to M_1^I.

Moving backwards, M uses this information and hence knows that if he opens with MD he will from period r^F onwards be '*rewarded'* by payoff a. But M also knows that such inducement play may be *costly,* payoff b, if F plays F_1^I. Therefore, for M to surely-win his reward must more than offset his cost, in which case M's optimal play in period 1 will be MD even if he knows with certainty that F_1^I will be played. Formally, the following incentive compatibility condition for $b(F_1^I) = \{M_1^D\}$ needs to hold

$$\underbrace{br^F}_{(MD,FI)} + \underbrace{a(r^M - r^F)}_{(MD,FD)} > \underbrace{dr^M}_{(MI,FI)} . \tag{6}$$

Rearranging (6) then yields

$$r^M(0) > \overline{r^M}(0) = \frac{a-b}{a-d} r^F \overset{(1)}{=} 1.5 r^F,^{22} \tag{7}$$

where the left-hand side (LHS) and right-hand side (RHS) report M's payoffs under F_1^I from playing MD and MI respectively. The $\overline{r^M}(0)$ variable is therefore the *necessary and sufficient* degree of M commitment that delivers M's sure-win for the case $R = 0$ - for both the general game and the specific game.

The rest of the proof in Appendix A derives such necessary and sufficient conditions for any values of r^M and r^F (and hence any type of asynchrony $R > 0$ and all nodes n^M), using a mathematical induction technique and the recursive scheme (3). It then combines them into the sufficient condition reported in (4). □

Intuitively, the fact that M is not willing to accommodate excessively expansionary F policy, and ready to contract the economy if necessary, eliminates the incentive of F to run structural deficits and accumulate debt. This result can be thought of as the case of dominant M policy of Sargent and Walace (1981); or Leeper's (1991) active M and passive F policy.

In contrast, the low commitment values $r^M < \overline{r^M}$ may provide an insufficiently long-lasting reward to compensate M for the initial cost, and hence fail to guarantee M's

[21] It will become evident that for most parameter values satisfying (4) there will be a *unique* Disciplined SPNE. Nevertheless, since our attention is on the equilibrium path we will not examine the exact number of SPNE (off-equilibrium behaviour).

[22] The right hand side of the $\overset{(1)}{=}$ notation reports the outcomes of the specific game in which (1) holds.

sure-win. This may or may not be the case since (4) reports a sufficient condition. Section 6 examines the parameter space $r^M < \overline{r^M}$ in more detail.

Note that for all types of asynchrony, R, and all general values of the payoffs satisfying (2), the threshold $\overline{r^M}$ is finite. It therefore follows that, under a patient M, a sufficient amount of M commitment that uniquely achieves the socially optimal outcomes (MD, FD) exists for any general payoffs.

Also note that, in contrast to the static concept of commitment, our framework gives us additional valuable information. Specifically, it tells us the exact degree of commitment that is required - as a function of several variables. The following Corollary, implied by (4), summarizes the various relationships:

Corollary 1. *The sufficient degree of M commitment* $\overline{r^M}$ *from Proposition 1, that ensures optimal long-run M policy outcomes and also disciplines F policy, is increasing in* r^F, d, v, y *and decreasing in* a, b, w, z.

As discussed above, the payoffs $\{a, b, d, v, w, y, z\}$ that determine the threshold value $\overline{r^M}$, and hence the policy outcomes, are functions of the deep parameters of the underlying macroeconomic model. That is, they depend on the players' preferences and the structure of the economy. Note that (i) the relationships are in line with conventional wisdom, and (ii) most parameters affect $\overline{r^M}$ in opposite directions for the two policymakers. For example, M's higher inflation cost reduces $\overline{r^M}$ (and makes it easier for M to surely-win), whereas F's higher inflation cost increases $\overline{r^M}$ (and makes it harder).

5. Results: Impatient Policymakers

In this section we consider the policymakers' discounting the future, and show that the qualitative nature of the results is unchanged. Nevertheless, several novel insights emerge under dynamic commitment that qualify the intuition of the standard static commitment concept. To separate the effects of F's and M's discounting we examine each in turn.

5.1. **F's Impatience.** This section shows that F's discounting may *weaken* the above sufficient conditions for M's sure-win, and hence improve coordination between the policies.

Proposition 2. *Consider the Battle of the sexes policy interaction in which* (2) *holds, and assume* $\delta_M = 1$, $0 \leq \delta_F \leq \overline{\delta_F} < 1$ *where* $\overline{\delta_F}$ *is some upper bound, and* $a > 2d - b$. *Then for M to surely-win and F to surely-lose it suffices that*

$$r^M > \overline{r^M} = r^F. \tag{8}$$

Proof. Appendix B shows that for some parameter values (including those of the specific Battle of the sexes game in (1)), the sufficient threshold is $\overline{r^M} = r^F$ and hence *any* $r^M > r^F$ uniquely ensures discipline of both policies. □

While it only applies under sufficiently impatient F, this is not entirely unrealistic as one would expect F's impatience to go hand in hand with F's ambition. Both are likely to be driven by the same political economy factors.

5.2. **M's Impatience.** This section shows that M's impatience strengthens the above sufficient condition, ie it makes it more difficult for M to surely-win and for the policymakers to coordinate on the socially optimal outcome. The intuition is similar to a standard repeated game in which it is harder to deter an impatient player from defecting since the future reward has a smaller present value.

Perhaps surprisingly, and in contrast to the standard commitment concept, it also shows that if a player is very impatient then even an infinitely strong commitment may be insufficient to ensure his sure-win. This is regardless of the other player's degree of patience.

Proposition 3. *Consider the Battle of the sexes policy interaction in which* (2) *holds and some threshold discount factor*

$$\overline{\delta_M} = \sqrt[r^F]{\frac{d-b}{a-b}} \stackrel{(1)}{=} \sqrt[r^F]{\frac{1}{3}}. \tag{9}$$

(i) If M is sufficiently patient, $\delta_M > \overline{\delta_M}$, then there exists $\overline{r^M} \in \mathbb{N}$ such that, for all $r^M > \overline{r^M}$ and $\forall r^F, R, \delta_F$, M surely-wins and F surely-loses.
(ii) If M is sufficiently impatient, $\delta_M < \overline{\delta_M}$, then even an infinitely strong M commitment, $r^M \to \infty$, does not ensure M to surely-win.

Proof. Note that (9) yields $0 < \overline{\delta_M} < 1$ for all assumed general values, which follows from $a > d > b$ in (2). For the details of the proof, see Appendix C. □

Claim (ii) of Proposition 3 stands in stark contrast to the static commitment concept in which the committed player uniquely ensures his preferred equilibrium in the Battle of the sexes regardless of his discounting, $\forall \delta_M$.

While Proposition 3 reports the *sufficient* bound $\overline{\delta_M}$, it does not provide the sufficient commitment level $\overline{r^M}$ - it only shows its existence. Nevertheless, since the proof of Proposition 1 showed that the $R = 0$ case is representative of the more asynchronous cases, we will investigate $\overline{r^M}(0)$ under impatience and extend our conclusions to the remaining R cases.[23]

Proposition 4. *Consider the Battle of the sexes policy interaction in which* (2) *holds and $R = 0$. The threshold $\overline{r^M}(0)$ is increasing in r^F and decreasing in δ_M, the latter implying that M's commitment and patience are substitutes in ensuring that M surely-wins.*

Proof. Appendix D shows that the necessary and sufficient M commitment level is

$$r^M > \overline{r^M}(0) = \log_{\delta_M}\left(\frac{a-b}{a-d}\delta_M^{r^F} - \frac{d-b}{a-d}\right) \stackrel{(1)}{=} \log_{\delta_M}\left(\frac{3}{2}\delta_M^{r^F} - \frac{1}{2}\right), \tag{10}$$

from which the implied necessary and sufficient patience threshold for M, $\overline{\delta_M}(0)$, is equal to the sufficient threshold $\overline{\delta_M}$ from (9). These thresholds are plotted in Figure 6 which demonstrates the claims graphically. For formal proofs see Appendix D. □

The following remark summarizes the above discussion with regard to the discounting.

[23] For example (19) in Appendix A shows that while the thresholds $\overline{r^M}(R)$ for $R \in (0, 1)$ may differ quantitatively from $\overline{r^M}(0)$, they are qualitatively the same.

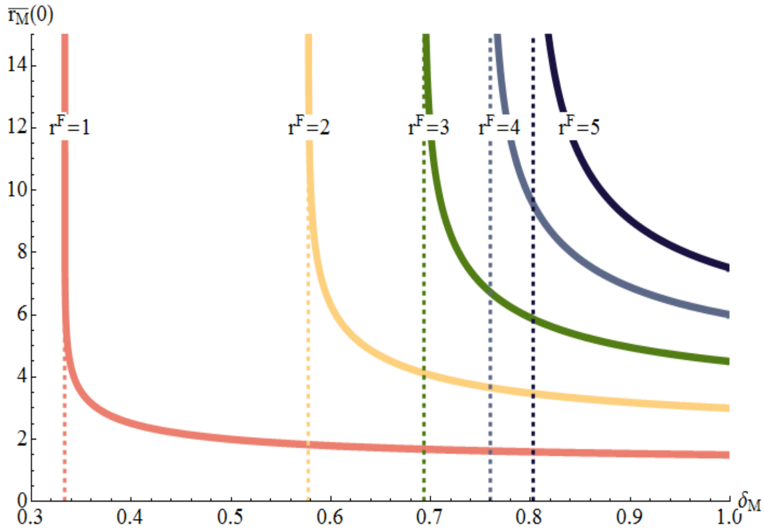

FIGURE 6. Dependence of $\overline{r^M}(0)$ on δ_M for various r^F (from (10) for the specific game (1)). Dotted asymptotes correspond to bounds $\overline{\delta_M}(0)$ for each particular r^F.

Remark 1. *Proposition 4 implies that (i) the results of Section 4 are robust to players' discounting; and that (ii) a less patient central banker needs to commit more strongly/explicitly to uniquely ensure his preferred long-term outcomes.*

6. Results: Insufficient M Commitment

Sections 4 and 5 focused on the situations in which M surely-wins the Battle, $r^M > \overline{r^M}$. To complement those results, this section briefly examines the outcomes under $r^M \leq \overline{r^M}$. It should now be apparent that all our previous results apply analogously.

Corollary 2. *Consider the Battle of the sexes policy interaction in which* (2) *holds. If*

(11) $$r^F > \overline{r^F}, \quad \textit{or equivalently,} \quad r^M < \underline{r^M},$$

where $\overline{r^F}$ and $\underline{r^M}$ are the 'mirror images' of $\overline{r^M}$ derived in Sections 4-5, then F surely-wins and M surely-loses.

Specifically, $\overline{r^F}$ is obtained from $\overline{r^M}$ by swapping all the corresponding variables and payoffs of players M and F. Conversely, $\underline{r^M}$ is some reciprocal of $\overline{r^M}$ that corresponds to (and is implied by) $\overline{r^F}$. For a graphical depiction of the r^M space featuring the thresholds see Figure 7.

For example in the specific Battle of the sexes, under patient policymakers M surely-wins if and only if $r^M(0) > \overline{r^M}(0) = \frac{3}{2}r^F$ (see (5)); whereas M surely-loses iff

$$r^F > \overline{r^F} = \frac{3}{2}r^M, \quad \textit{or equivalently,} \quad r^M < \underline{r^M} = \frac{2}{3}r^F.$$

Since the payoffs of the specific game are symmetric, the thresholds remain the same not only qualitatively but also quantitatively. Recall from Proposition (3) that $\overline{r^M}$ - and

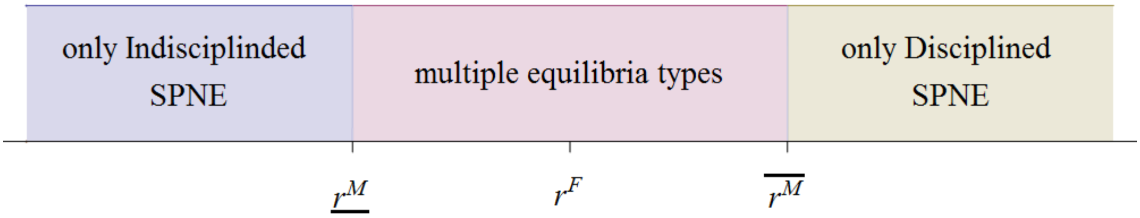

Figure 7. The r^M space featuring the thresholds and regions of SPNE.

hence $\overline{r^F}$ and $\underline{r^M}$ - only exist if the more committed policymaker is sufficiently patient. Again, the threshold patience level is a mirror image of $\overline{\delta_M}$. For example, the analog of the threshold from (9), $\overline{\delta_M} = \sqrt[r^F]{\frac{d-b}{a-b}} \overset{(1)}{=} \sqrt[r^F]{\frac{1}{3}}$, is

$$\overline{\delta_F} = \sqrt[r^M]{\frac{v-w}{z-w}} \overset{(1)}{=} \sqrt[r^M]{\frac{1}{3}}.$$

Let us use Figure 7 to summarize the above results. The $\frac{r^M}{r^F}$ space can be broken into three main regions, in which there are: (i) only the Disciplined type of SPNE, (ii) only the Indisciplined type of SPNE, (iii) multiple types of SPNE. Under the conditions of Proposition 3(ii) the threshold $\overline{r^M}$ does not exist so there are either two regions, or just one region if $\overline{r^F}$ does not exist also.

It is straightforward to show that in the intermediate region $r^M \in \left[\underline{r^M}, \overline{r^M}\right]$ there are either (i) both Disciplined and Indisciplined SPNE, or (ii) only one of these two types, or (iii) neither of them. We leave a thorough investigation of this region (in which repetition of the game needs to be taken into account) for future research. Let us just report one finding.

Proposition 5. *Consider the Battle of the sexes policy interaction in which* (2) *holds and* $r^M \in \left(r^F, \overline{r^M}\right)$. *Despite* M *being the more strongly committed player, there exist parameter values under which* M *likely-loses and* F *likely-wins.*

Proof. See Appendix E. □

The fact that the player with a stronger commitment (note $r^M > r^F$) will be more 'likely' to lose contrasts with the standard commitment solution, in which the committed player *always* gets its preferred outcome in the Battle of the sexes.

Intuitively, in the example in the proof it occurs because M is quite sensitive to the output cost, which can be interpreted as insufficiently conservative placing high weight on avoiding a conflict with F policy. The high mis-coordination cost would therefore discourage him from disinflating. The novel insight is that insufficient M conservatism may reduce the effectiveness of M commitment.

7. Heterogeneous Fiscal Policy in a Monetary Union

An advantage of our game theoretic approach with generalized commitment is that we can extend our analysis to any number of players. To demonstrate, let us examine the case in which F is heterogeneous, ie there are various F policymakers of potentially different economic size (influence/importance), and with differing degrees of commitment. This arguably describes the situation in the European Union, and the US to some extent, with a common currency and hence common M policy, but independent F policies.

The players' set is then $I = \{M, F^j\}$ where $j \in [1, J]$ denotes a certain country, r_j^F denotes this country's degree of F commitment, and w_j denotes this country's relative economic size such that $\sum_{j=1}^{J} w_j = 1$. We find it natural to focus on these two types of F heterogeneity. Let us assume that the payoff of M *overall* is a weighted average - with weights w_n - of the banker's payoffs gained from the interaction with each F^j.

Let us depict the simple case with patient players in which $\delta_F^j = \delta_M = 1, \forall j$. Furthermore, let us focus on $R = 0$, which was shown to be representative of the more asynchronous cases, and which is re-defined under heterogeneous F as $R_j = 0, \forall j \in I$ - that is $\frac{r^M}{r_j^F} = \lfloor \frac{r^M}{r_j^F} \rfloor, \forall j \in I$.

Remark 2. *The nature of the results under homogenous F policy remains unchanged under heterogeneous F policies. For example, the necessary and sufficient condition of the Battle of the sexes under patient players and* $R = 0$ *generalizes from equation* (7) *in Appendix A, namely* $r^M(0) > \overline{r^M}(0) = \frac{a-b}{a-d} r^F \overset{(1)}{=} \frac{3}{2} r^F$, *to*

$$r^M(0) > \overline{r^M}(0) = \frac{a-b}{a-d} \sum_{j=1}^{J} w_j r_j^F \overset{(1)}{=} \frac{3}{2} \sum_{j=1}^{J} w_j r_j^F. \tag{12}$$

The sufficient conditions are modified analogously.

Proof. See Appendix F. □

For example, with two countries X and Y, the former being double the size of the latter, $w_X = 2w_Y = \frac{2}{3}$, the condition in (12) for the specific Battle of the sexes game becomes $r^M(0) > \overline{r^M}(0) = r_X^F + \frac{1}{2} r_Y^F$.[24]

It is important to note that our analysis allows for modelling a potential moral hazard on the part of individual governments. This can arise since the benefits of the F stimulus may accrue more to the fiscally indisciplined country, whereas the costs in terms of tighter M policy are spread across all countries (see eg Masson and Patillo (2002)).

The smaller the extent to which they internalize the cost borne by other members, the higher the parameter output cost $(a - b)$ - the M policymaker incurs a great disutility from having to fight the governments more aggressively. Therefore, the sufficient degree of M commitment $\overline{r^M}$ that offsets this moral hazard problem increases as shown in (12). Nevertheless, the same policy implication still applies, namely that to (attempt to) discourage F indiscipline by members, a stronger M commitment must be implemented.[25]

[24] Libich and Stehlík (2009) calibrate a different (stochastic) model with the European Monetary Union data and provide a quantitate estimate of the required degree of commitment of the European Central Bank.

[25] It may however be the case in a monetary union featuring moral hazard that $v < w$, under which FI is the government's strictly dominant strategy in the static game, and hence even a patient central bank with an infinitely strong M commitment does not uniquely secure a sure-win under such circumstances. This implies that other, more direct types of enforcement/punishment mechanisms may have to be used to discourage member countries from F indiscipline.

8. Case Study of New Zealand by Dr Don Brash[26]

'New Zealand provides an interesting case study illustrating the arguments in the article. We adopted a very strong commitment by the monetary authority, the Reserve Bank of New Zealand, when the Minister of Finance signed the first Policy Targets Agreement (PTA) with me as Governor under the new Reserve Bank of New Zealand Act 1989 early in 1990. The PTA required me to get inflation as measured by the CPI to between 0 and 2% per annum by the end of 1992, with the Act making it explicit that I could be dismissed for failing to achieve that goal unless I could show extenuating circumstances in the form, for example, of a sharp increase in international oil prices. At the time, inflation was running in excess of 5%.

In the middle of 1990, the Government, faced with the prospect of losing an election later in the year, brought down an expansionary budget. I immediately made it clear that this expansionary fiscal policy required firmer monetary conditions if the agreed inflation target was to be achieved, and monetary conditions duly tightened.

Some days later, an editorial in the "New Zealand Herald", New Zealand's largest daily newspaper, noted that New Zealand political parties could no longer buy elections because, when they tried to do so, the newly instrument-independent central bank would be forced to send voters the bill in the form of higher mortgage rates.

I was later told by senior members of the Opposition National Party that the Bank's action in tightening conditions in response to the easier fiscal stance had had a profound effect on thinking about fiscal policy in both major parties in Parliament.

Some years later, in 1996, the Minister of Finance of the then National Party Government announced that he proposed to reduce personal income tax rates subject to this being consistent with the Government's debt to GDP target being achieved, to the fiscal position remaining in surplus, and to the fiscal easing not requiring a monetary policy tightening. The Minister formally wrote to me asking whether tax reductions of the kind proposed would under the economic circumstances then projected, require me to tighten monetary conditions. Given how the Bank saw the economy evolving at that time, I was able to tell the Minister that tax reductions of the nature he proposed would not require the Bank to tighten monetary conditions in order to stay within the inflation target.'

9. Summary and Conclusions

The long-run stance of fiscal policy in a number of countries (including the US and EU) has raised concerns about the degree of discipline, and about the risks for the credibility and outcomes of monetary policy. This paper highlights the importance of

[26]Governor of the Reserve Bank of New Zealand during 1988-2002, in which period the Bank pioneered its explicit inflation targeting framework.

understanding the monetary-fiscal interactions, and the effect of various commitment arrangements in assessing whether this poses a problem.

Specifically, to contribute to this debate we use a novel asynchronous game theory framework to enrich the standard commitment concept in a number of respects. Most importantly, it is dynamic and hence allows for: (i) concurrent commitment of more than one player/policy, (ii) partial commitment, and (iii) endogenously determined (optimally selected) commitment.

Our analysis shows that the effect of commitment on economic outcomes of the policy interaction crucially depends on its strength relative to the degree of fiscal rigidity and ambition. Most interestingly, it is shown that a sufficiently strong monetary commitment can not only resist fiscal pressure coming from excessively ambitious governments, but also indirectly 'discipline' those governments, and hence achieve socially desirable outcomes for *both* policies. Our contribution to the literature initiated by Sargent and Wallace (1981) and Leeper (1991) therefore lies in formally identifying a policy variable that can tip the balance of 'power' between the policies.[27]

The implication for monetary policymakers (in countries with persistently ambitious fiscal policymakers) is that to discourage and/or counter-act over-expansionary fiscal policies, they should make their low-inflation target *more explicit* in their statutes. The implication for fiscal policymakers is that imposing monetary commitment (eg legislating a numerical long-run inflation target) may provide a way to indirectly tie their hands if direct fiscal reform seems politically infeasible.

It is important to note that the proposed dynamic commitment is to the regime itself and hence to long-run outcomes, rather than to specific short-run policies or rules within that regime. It therefore still allows for deviations from the long-run objectives in the short-run, and does not restrict the policymakers' flexibility to respond to various shocks or stabilize the real economy. Such short-run stabilization can be done in any fashion. In that our long-term commitment is compatible with the timeless perspective commitment of Woodford (1999), or quasi commitment of Schaumburg and Tambalotti (2007), as well as discretion.

Our companion work examines several alternative specifications of commitment (in different macro and microeconomic contexts), such as *stochastic commitment* or *time-varying commitment*, as well as *endogenous commitment*, and implies that the intuition of all these settings is comparable (LHHS (2007)). Nevertheless, more research is required to get a deeper iunderstanding of the complex effects of various real world commitment arrangements, especially in the light of the interactions between M and F policies.

10. References

ADAM, K., and R. BILLI (2008): "Monetary Conservatism and Fiscal Policy," Journal of Monetary Economics, 55(8), 1376-1388.

[27]We have assumed throughout that the central bank is responsible and the government ambitious. Naturally, in the unlikely case of an ambitious central bank and a responsible goverment the conclusions are *reversed*, ie it is required that F policy is sufficiently strongly committed relative to M policy. If both policies are responsible than there exists no structural deviation from the socially optimal long-term outcomes. Nevertheless, can can be argued that because the government's ambition in the real world may change over time with the political cycle, implementing a sufficiently explicit M commitment acts as a 'credibility insurance' to M policy outcomes.

ALESINA, A., and G. TABELLINI (1987): "Rules and Discretion with Noncoordinated Monetary and Fiscal Policies," Economic Inquiry, 25(4), 619 - 630.

ARTIS, M. J., and B. WINKLER (1998): "The Stability Pact: Trading Off Flexibility for Credibility?," in Fiscal Aspects of European Monetary Integration, ed. by Hughes Hallett, Hutchison, and Jensen: Cambridge University Press, 157-188.

AUMANN, R., and S. SORIN (1989): "Cooperation and Bounded Recall," Games and Economic Behaviour, 1, 5-32.

BACKUS, D., and J. DRIFFILL (1985): "Inflation and Reputation," The American Economic Review, 75, 530-538.

BARNETT, R. (2001): "Inflation, Taxes, and the Coordination of Monetary and Fiscal Policy by Use of a Game of Chicken," Canadian Journal of Economics, 34, 82-99.

BARRO, R. J., and D. B. GORDON (1983): "A Positive Theory of Monetary Policy in a Natural Rate Model," Journal of Political Economy, 91, 589-610.

BENHABIB, J., and S. EUSEPI (2005): "The Design of Monetary and Fiscal Policy: A Global Perspective," Journal of Economic Theory, 123(1), 40-73.

BERNANKE, B. S., T. LAUBACH, F. S. MISHKIN, and A. POSEN (1999): Inflation Targeting: Lessons from the International Experience. Princeton: Princeton University Press.

BHATTACHARYA, J., and J. H. HASLAG (1999): "Monetary Policy Arithmetic: Some Recent Contributions," Economic and Financial Policy Review, Federal Reserve Bank of Dallas, Q III, 26-36.

BLAKE, A. P., and M. WEALE (1998): "Costs of Separating Budgetary Policy from Control of Inflation: A Neglected Aspect of Central Bank Independence," Oxford Economic Papers, 50(3), 449-67.

BOHNER, M., and A. PETERSON (2001): Dynamic Equations on Time Scales. Birkhauser Boston, 704 Inc., Boston, MA, 2001.

BRANCH, W. A., T. DAVIG, and B. MCGOUGH (2008): "Monetary–Fiscal Policy Interactions under Implementable Monetary Policy Rules," Journal of Money, Credit and Banking 40(5).

CALVO, G. A. (1983): "Staggered Prices in a Utility-Maximizing Framework," Journal of Monetary Economics, 12, 383-98.

CHADHA, J. S., and C. NOLAN (2007): "Optimal Simple Rules for the Conduct of Monetary and Fiscal Policy " Journal of Macroeconomics, 29(4), 665-689.

CHO, I., and A. MATSUI (2005): "Time Consistency in Alternating Move Policy Games," Japanese Economic Review, 56(3) 273–294. .

COCHRANE, J. H. (2009): "Fiscal Theory, and Fiscal and Monetary Policy in the Financial Crisis," unpublished.

COOPER, R. N. (1969): "Macroeconomic Policy Adjustment in Interdependent Economies " The Quarterly Journal of Economics, 83, No. 1, 1-24.

DIXIT, A., and L. LAMBERTINI (2001): "Monetary–Fiscal Policy Interactions and Commitment Versus Discretion in a Monetary Union " European Economic Review, 45(4), 977-987.

DIXIT, A., and L. LAMBERTINI (2003): "Interactions of Commitment and Discretion in Monetary and Fiscal Policies," American Economic Review, 93(5), 1522-1542.

EUSEPI, S., and B. PRESTON (2008): "Stabilizing Expectations under Monetary and Fiscal Policy Coordination," NBER Working Papers 14391, National Bureau of Economic Research, Inc.

GALI, J., and T. MONACELLI (2005): "Optimal Fiscal Policy in a Monetary Union," Proceedings, Federal Reserve Bank of San Francisco.

GERAATS, P. M. (2002): "Central Bank Transparency," Economic Journal, 112, F532-65.

GOODFRIEND, M. (2003): "Inflation Targeting in the United States?," National Bureau of Economic Research Inc NBER Working Papers: 9981.

HUGHES HALLETT, A., and J. LIBICH (2007): "Fiscal-Monetary Interactions: The Effect of Fiscal Restraint and Public Monitoring on Central Bank Credibility," Open Economies Review, 18 (5), 559-576.

HUGHES HALLETT, A., and D. N. WEYMARK (2005): "Independence before Conservatism: Transparency, Politics and Central Bank Design," German Economic Review, 6, 1-21.

LEEPER, E. M. (1991): "Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies," Journal of Monetary Economics 27 (1), 129-147.

LIBICH, J., A. HUGHES HALLETT, and P. STEHLÍK (2007): "Monetary and Fiscal Policy Interaction with Various Degrees and Types of Commitment," CEPR DP 6586.

LIBICH, J., and P. STEHLÍK (2009): "Monetary Policy Facing Fiscal Indiscipline under Generalized Timing of Actions," under review

LOHMANN, S. (1992): "Optimal Commitment in Monetary Policy: Credibility Versus Flexibility," American Economic Review, 82, 273-86.

MASKIN, E., and J. TIROLE (1988): "A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition with Large Fixed Costs," Econometrica, 56, 549-69.

MASSON, P., and C. PATTILLO (2002): "Monetary Union in West Africa: An Agency of Restraint for Fiscal Policies?," Journal of African Economies., 11 387-412.

MUNDELL, R. (1962): "Appropriate Use of Monetary and Fiscal Policy for Internal and External Stability," IMF Staff Papers, 9, 70-79.

NORDHAUS, W. (1994): "Policy Games: Coordination and Independence in Monetary and Fiscal Policies," Brookings Papers on Economic Activity, 1994:2, 139-216.

PERSSON, M., T. PERSSON, and L. E. O. SVENSSON (2006): "Time Consistency of Fiscal and Monetary Policy: A Solution," Econometrica, 74(1), 193-212.

PETIT, M. L. (1989): "Fiscal and Monetary Policy Co-Ordination: A Differential Game Approac H," Journal of Applied Econometrics, 4(2), 161-79.

RESENDE, C., and N. REBEI (2008): "The Welfare Implications of Fiscal Dominance," Bank of Canada Working Paper 2008-28.

ROGOFF, K. (1985): "The Optimal Degree of Commitment to an Intermediate Monetary Target," Quarterly Journal of Economics, 100, 1169-89.

SARGENT, T. J. (1991): "Equilibrium with Signal Extraction from Endogenous Variables," Journal of Economic Dynamics and Control, Elsevier, vol. 15(2), 245-273.

SARGENT, T. J., and N. WALLACE (1981): "Some Unpleasant Monetarist Arithmetic," Federal Reserve Bank of Minneapolis Quarterly Review, 5, 1-17.

SCHAUMBURG, E., and A. TAMBALOTTI (2007): "An Investigation of the Gains from Commitment in Monetary Policy," Journal of Monetary Economics, 54(2), 302-324.

SIMS, C. (1994): "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," Economic Theory, 4(3), 381-99.

TAYLOR, J. B. (1979): "Staggered Wage Setting in a Macro Model," American Economic Review, 69, 108-13.

TAYLOR, J. B. (1993): "Discretion Versus Policy Rules in Practice," Carnegie-Rochester Conference Series on Public Policy, 39, 195-214.

TINBERGEN, J. (1954): Centralization and Decentralization in Economic Policy. Amsterdam.

TOBIN, J. (1982): "Money and Finance in the Macroeconomic Process," Journal of Money, Credit and Banking, 14 (2), 171-204.

VAN AARLE, B., J. ENGWERDA, and J. PLASMANS (2002): "Monetary and Fiscal Policy Interaction in the Emu: A Dynamic Game Approach " Annals of Operations Research, 109(1-4), 229-264.

WALSH, C. E. (1995): "Optimal Contracts for Central Bankers," American Economic Review, 85, 150-67.

WEN, Q. (1994): "The Folk Theorem for Repeated Games with Complete Information," Econometrica, 62, 949-954.

WOODFORD, M. (1994): "Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy," Economic Theory, 4(3), 345-80.

WOODFORD, M. (1999): "Optimal Monetary Policy Inertia," Manchester School, 67, 1-35.

WYPLOSZ, C. (1991): "Monetary Union and Fiscal Policy Discipline," CEPR DP488.

Appendix A. Proof of Proposition 1

Proof. We continue the proof in the main text, to examine the case of $R > 0$.

B) $\mathbf{n^M = N^M}$ **under** $\mathbf{R > 0}$: From Definition 2 it follows that the number of M's moves is $N^M = \frac{T(r^M, r^F)}{r^M} > 1$. A condition analogous to (6) is the following

$$\underbrace{br^F R}_{(MD,FI)} + \underbrace{a(r^M - r^F R)}_{(MD,FD)} > \underbrace{dr^M}_{(MI,FI)} . \tag{13}$$

Rearrange this to obtain

$$r^M > \frac{a-b}{a-d} Rr^F, \tag{14}$$

This means that, if (14) holds, a patient M will find it optimal to play M_N^D for all histories.

C) $\mathbf{n^M+1 \rightarrow N^M}$ (if applicable, ie if $1 \le n^M < N^M$): The proof proceeds by induction. We first assume that M's unique best play in the (n^M+1)-th step is MD regardless of F's preceding play (ie that M_{n+1} is history-independent), and we attempt to prove that this implies the same assertion for the n^M-th step. Intuitively, this means that if M inflates he finds it optimal to immediately disinflate. Two scenarios are possible in terms of the underlying F behaviour since that will determine the costs of the disinflation. If F runs a deficit, F^I, the payoffs b and w will occur for at least one period, whereas if F runs a balanced budget, F^D, the disinflation will only be accompanied by the payoffs a and v (note that in the former case the disinflation is more *costly* to both policymakers since $a > b$ and $v > w$ from (2)). This implies that one of the following two conditions, analogous to (6), will apply at any move n^M

$$bk_n + a(r^M - k_n) + a[r^F - (r^F - k_{n+1})] > dr^M + b[r^F - (r^F - k_{n+1})], \tag{15}$$

$$bk_n + a(r^M - k_n) > d[r^M - (r^F - k_{n+1})] + a(r^F - k_{n+1}). \tag{16}$$

Which of these two conditions is relevant to a certain n^M depends on F's payoffs $\{v, w, y, z\}$, and importantly on k_{n+1}. Specifically, if

$$\underbrace{(r^F - k_{n+1})z}_{(MI,FI)} + \underbrace{k_{n+1}w}_{(MD,FI)} \ge \underbrace{(r^F - k_{n+1})y}_{(MI,FD)} + \underbrace{k_{n+1}v}_{(MD,FD)}, \tag{17}$$

then (15) obtains, otherwise (16) is the relevant condition.[28]

Now, we will show that if the conditions (15) and (16) are satisfied at $n^M = 1$, then they hold in all other n^M as well. This convenient feature notably simplifies the solution of the game.

Lemma 1. *Consider the Battle of the sexes policy interaction in which* (2) *holds and* $\delta_F = \delta_M = 1$. *For a given* R, *out of the necessary and sufficient conditions for* M *to surely-win,* $\{M_n^D\} = b(F^I)$, *the one regarding* $n^M = 1$ *yields as least as high* $\overline{r^M}(R)$ *as any other* n^M. *Therefore,* $\{M_1^D\} = b(F_1^I)$ *is the sufficient condition.*

Proof. Equations (15) and (16) can be, respectively, rearranged into

$$r^M > \frac{a-b}{a-d}(k_n - k_{n+1}) \quad \text{and} \quad r^M > \frac{(a-b)k_n}{a-d} + (r^F - k_{n+1}). \tag{18}$$

The strength of both conditions is increasing in k_n and decreasing in k_{n+1}. Thus the strongest condition is guaranteed by the maximum of $(k_n - k_{n+1})$. From (3) it follows that $k_n - k_{n+1} \le Rr^F$. The fact that $k_1 - k_2 = Rr^F$ then proves the claim. □

Continuing the proof of Proposition 1, this property means that regardless of the exact dynamics/asynchrony, it suffices to focus on the initial simultaneous move (similarly to a one-shot game) *assuming* that all further relevant conditions hold. If the strongest condition for $n^M = 1$ is satisfied we then know that a unique (type of) equilibrium outcome obtains throughout. Using the implied $k_1 = r^F$ and $k_{n+1} = k_n - Rr^F$ jointly

[28] For the specific game this condition becomes $k_{n+1} \le \frac{z-y}{z-y+v-w} \stackrel{(1)}{=} \frac{4}{5}r^F$. This implies that in the game in Figure 5 with $r^F = 3$ and $r^M = 5$, all disinflations (in $n^M \ge 2$) would be costly and (15) would apply. We will see below that the parameter space under which (16) obtains gets smaller with the F's impatience.

yields $k_2 = (1-R)r^F$. Substituting these into (15)-(16) or (18) we obtain, together with (7)

$$(19) \quad r^M > \overline{r^M}(R) = \begin{cases} \frac{a-b}{a-d} r^F \overset{(1)}{=} \frac{3}{2} r^F & \text{if } R = 0, \\ \frac{a-b}{a-d} R r^F \overset{(1)}{=} \frac{3}{2} R r^F & \text{if } R > \bar{R} = \frac{v-w}{z-y+v-w} \overset{(1)}{=} \frac{1}{5}, \\ \left(\frac{a-b}{a-d} + R\right) r^F \overset{(1)}{=} \left(\frac{3}{2} + R\right) r^F & \text{if } R \leq \bar{R} = \frac{v-w}{z-y+v-w} \overset{(1)}{=} \frac{1}{5}, \end{cases}$$

where the threshold $\bar{R} \in (0,1)$ is implied by (17). These inequalities are the three necessary and sufficient conditions for uniqueness of the Disciplined type of SPNE (note that all three are at least as strong as the condition for N^M in (14)). Combining the three conditions implies the sufficient conditions (4) and (5), and completes the proof of Proposition 1. □

Appendix B. Proof of Proposition 2

Proof. Under $R = 0$ the value of δ_F does not affect the relevant sufficient condition in (7). However, if $R = (0,1)$ and F is sufficiently impatient, $\delta_F \leq \overline{\delta_F}$, where the threshold value $\overline{\delta_F}$ is a function of $\{r^M, r^F, v, w, y, z\}$, the sufficient condition will alter. Instead of deriving analytically $\overline{\delta_F}$ from (17) we focus on the extreme case $\overline{\delta_F} = \delta_F = 0$ which is a sufficiently low threshold for all r^M, r^F and for all $\{v, w, y, z\}$ satisfying (2).

The impatient F will disregard the future and always play $F_t^* \in b(F_t)$. Since, for all but the initial move the policymakers never move simultaneously, this implies $F_{n>1}^* \in b(M_{t-1})$. Intuitively, a sufficiently impatient F will never reduce G before the start of disinflation and hence disinflation will always be costly for both players. Formally, (16) no longer applies and (15) becomes the relevant condition $\forall n^M, R \in (0,1)$, and for all $\{a, b, c, d, v, w, y, z\}$ satisfying (2).

Hence we need to show that any $r^M > r^F$ satisfy the following two conditions: (i) under $R = 0$ it holds that $\frac{r^M}{r^F} > \frac{a-b}{a-d}$ (from (7)) and (ii) $\forall R \in (0,1)$ it is true that $\frac{r^M}{r^F} > \frac{a-b}{a-d} R$ (from (19)). To prove (ii) note that any $r^M > r^F$ has, from the definition of R, the property that $\frac{r^M}{r^F} \geq 1 + R$. Therefore claim (ii) can be rewritten as $1 + R > \frac{a-b}{a-d} R$. Divide both sides by R to obtain $\frac{1}{R} + 1 > \frac{a-b}{a-d}$. To see that this is satisfied we utilize two facts. First, $\frac{1}{R} + 1 > 2$ since $R < 1$. Second, rearrange $a > 2d - b$ into $2 > \frac{a-b}{a-d}$. Combining these gives $\frac{1}{R} + 1 > 2 > \frac{a-b}{a-d}$ which completes the proof of (ii). To show (i), note that under $R = 0$ all $r^M > r^F$ satisfy $\frac{r^M}{r^F} \geq 2$. Using this jointly with $2 > \frac{a-b}{a-d}$ completes the proof. □

Appendix C. Proof of Proposition 3

Let us first extend the result of Lemma 1 under impatience.

Lemma 2. *Consider the Battle of the sexes policy interaction in which* (2) *holds. Then* $\forall \delta_M, \delta_F$, *and* R, *out of the necessary and sufficient conditions for* M *to surely-win,* $\{M_n^D\} = b(F^I)$, *the one regarding* $n^M = 1$ *yields as least as high* $\overline{r^M}(R)$ *as any other* n^M. *Therefore,* $\{M_1^D\} = b(F_1^I)$ *is the sufficient condition.*

Proof. Lemma 1 shows this claim to hold under $\delta_M = \delta_F = 1$. The proof of Proposition 2 showed that δ_F affects whether (15) or (16) applies in some n^M, but not the implication of (18) that they are both the strongest at $n^M = 1$. Let us therefore consider the effect of M's impatience. Under $\delta_M < 1$, the inequality in (15) that applies to the case of $R > \bar{R}$, becomes

$$b\sum_{t=1}^{k_n}\delta_M^{t-1} + a\sum_{t=k_n+1}^{r^M}\delta_M^{t-1} + a\sum_{t=r^M+1}^{r^M+k_{n+1}}\delta_M^{t-1} > d\sum_{t=1}^{r^M}\delta_M^{t-1} + b\sum_{t=r^M+1}^{r^M+k_{n+1}}\delta_M^{t-1}. \tag{20}$$

This can be rearranged into

$$(d-b)\sum_{t=1}^{k_n}\delta_M^{t-1} - (a-d)\sum_{t=k_n+1}^{r^M}\delta_M^{t-1} - (a-b)\sum_{t=r^M+1}^{r^M+k_{n+1}}\delta_M^{t-1} < 0.$$

Use $a - b = (a-d) + (d-b)$ and split the first series to obtain

$$(a-b)\sum_{t=1}^{k_n}\delta_M^{t-1} - (a-d)\sum_{t=1}^{r^M}\delta_M^{t-1} - (a-b)\sum_{t=r^M+1}^{r^M+k_{n+1}}\delta_M^{t-1} < 0.$$

Now add $\sum_{t=k_n+1}^{r^M}\delta_M^{t-1}$ to both sides and collect the terms to get

$$(a-b)\sum_{t=1}^{r^M}\delta_M^{t-1} - (a-d)\sum_{t=1}^{r^M}\delta_M^{t-1} < (a-b)\delta_M^{k_n}\frac{1-\delta_M^{r^M+k_{n+1}-k_n}}{1-\delta_M}.$$

Since $\delta_M < 1$ we see that, analogously to Lemma 1, the strength of the condition is increasing in k_n and decreasing in k_{n+1}. Hence the same argument applies. We can also see that, for $R \leq \bar{R}$ (using (16)), the effect of M's impatience is analogous. Finally, for $R = 0$ we have $N^M = 1$ which finishes the proof. □

Now we complete the proof of Proposition 3 using Lemma 2.

Proof. Claim (i): It is apparent in (19) that the *strongest* possible necessary and sufficient condition (highest $\overline{r^M}(R)$) obtains under *costless* disinflation if F's payoffs $\{v, w, y, z\}$ are such that $\bar{R} \to 1$ (since the inflation cost d lasts the shortest period of time). Furthermore, we have shown in Proposition 2 that the opponent's impatience weakens the sufficient conditions. Therefore, we can focus on the analog of (15) under $0 \leq \delta_M < 1 = \delta_F$, which is

$$b\sum_{t=1}^{k_n}\delta_M^{t-1} + a\sum_{t=k_n+1}^{r^M}\delta_M^{t-1} > d\sum_{t=1}^{r^M-r^F+k_{n+1}}\delta_M^{t-1} + a\sum_{t=r^M-r^F+k_{n+1}+1}^{r^M}\delta_M^{t-1}. \tag{21}$$

Now use the fact that this condition is the strongest for $k_n = r^F$ and $k_{n+1} \to 0$ (the latter follows from $\bar{R} \to 1$), and rearrange to obtain

$$(a-d)\sum_{t=r^F+1}^{r^M-r^F}\delta_M^{t-1} > (d-b)\sum_{t=1}^{r^F}\delta_M^{t-1}. \tag{22}$$

It therefore suffices to show that the condition of Proposition 3, namely (9), implies (22). To do so note that (9) can be rearranged into $\delta_M^{r^F} > \frac{d-b}{a-b}$,which can be manipulated to give

$$0 < 1 - \frac{d-b}{a-d}\frac{1-\delta_M^{r^F}}{\delta_M^{r^F}}.$$

Since $\delta_M^{2r^F} > 0$, it is true that

$$0 < \delta_M^{2r^F}\left(1 - \frac{d-b}{a-d}\frac{1-\delta_M^{r^F}}{\delta_M^{r^F}}\right).$$

Consequently, for each $\delta_M = (0,1)$ there exists $\overline{r^M} \in \mathbb{N}$ such that for all $r^M > \overline{r^M}$

$$\delta_M^{r^M} < \delta_M^{2r^F}\left(1 - \frac{d-b}{a-d}\frac{1-\delta_M^{r^F}}{\delta_M^{r^F}}\right).$$

Multiplying both sides by $-(a-d)\delta_M^{r^F} > 0$ and dividing by $\delta_M^{2r^F}(1-\delta_M)$ we obtain

$$(a-d)\delta_M^{r^F}\frac{\left(1-\delta_M^{r^M-2r^F}\right)}{(1-\delta_M)} > (d-b)\frac{(1-\delta_M^{r^F})}{(1-\delta_M)}.$$

Note that the two fractions are in fact partial sums of geometric series with quotient δ_M, which is the desired condition in (22).

Claim (ii): It is apparent in (19) that the *weakest* possible necessary and sufficient condition (lowest $\overline{r^M}(R)$) obtains under *costly* disinflation if F's payoffs $\{v, w, y, z\}$ are such that $\bar{R} \to 0$ (since the disinflation cost b lasts the longest period of time). Furthermore, we have shown in Proposition 2 that the opponent's impatience weakens the sufficient conditions. Therefore, we can focus on the analog of (15) under $0 = \delta_F \leq \delta_M < 1$, (20), imposing the implication of Lemma 2 that $k_{n+1} = k_2 \to k_n = k_1 = r^p$ (the latter leading to $\bar{R} \to 0$). Substituting this into (20) yields

$$(a-d)\sum_{t=r^F+1}^{r^M}\delta_M^{t-1} + (a-b)\sum_{t=r^M+1}^{r^M+r^F}\delta_M^{t-1} > (d-b)\sum_{t=1}^{r^F}\delta_M^{t-1}. \tag{23}$$

Using the formula for a finite sum of geometric series and rearranging yields

$$(1-\delta_M^{r^F})[(a-b)\delta_M^{r^F} - (d-b)] \overset{(1)}{=} (1-\delta_M^{r^F})(2\delta_M^{r^F} - 1) > 0.$$

This is *not* satisfied for any values $\delta_M \leq \overline{\delta_M}$ where the threshold is from (9) (the fact that there may be no Disciplined SPNE implies that the inequality in (9) is strict). □

Appendix D. Proof of Proposition 4

Proof. Under M's impatience, the condition analogous to (6) becomes

$$b\sum_{t=1}^{r^F}\delta_M^{t-1} + a\sum_{t=r^F+1}^{r^M}\delta_M^{t-1} > d\sum_{t=1}^{r^M}\delta_M^{t-1}. \tag{24}$$

which can, using the formula for a sum of a finite series, be rewritten as

$$b\frac{1-\delta_M^{r^F}}{1-\delta_M}+a\delta^{r^F}\frac{1-\delta_M^{r^M-r^F}}{1-\delta_M}>d\frac{1-\delta_M^{r^M}}{1-\delta_M}.$$

By analyzing this equation we observe that (24) holds if and only if (10) is satisfied. Now we can utilize two properties which follow from (10). First, the argument of the logarithm in (10) is positive if and only if $\delta_M > \overline{\delta_M}$ holds (from (9)). Second, both the base and the argument of the logarithm in (10) lie strictly between 0 and 1. To see this, recall that

$$0<\frac{a-b}{a-d}\delta_M^{r^F}-\frac{d-b}{a-d}<\frac{a-b}{a-d}-\frac{d-b}{a-d}=1.$$

Therefore $\overline{r^M}(0)$ is positive and increasing in r^F. In order to prove the substitutability claim, take (10) and rewrite it as

$$\overline{r^M}=\frac{\ln(\frac{a-b}{a-d}\delta_M^{r^F}-\frac{d-b}{a-d})}{\ln\delta_M}.$$

Our task now is to show that $\overline{r^M}$ is decreasing in δ_M on the considered domain

$$D:=\left(\sqrt[r^F]{\frac{d-b}{a-b}},1\right).$$

This is shown in LHHS (2007). □

Appendix E. Proof of Proposition 5

Proof. To prove these (non)-existence claims it suffices to provide specific examples. For the reader's convenience we will consider the specification of Figure 2, namely $r^M = 5, r^F = 3$, and the values of the specific Battle of the sexes game (1) with only one change: the cost of disinflation for M, payoff b, will be made greater, $b = -3$. Let us first show that there exists no Disciplined SPNE and then that an Indisciplined SPNE exists.

Focus on the condition for M's last move, $n^M = N^M$, to be uniquely MD in equation (14), $\frac{r^M}{r^F} > \frac{a-b}{a-d}R$. Notice that, under these specific circumstances, this condition is not satisfied, $\frac{5}{3} \ngtr 2$. Therefore, M_3 is no longer history-independent and M_3 will be the best response to F's preceding move, F_4. Moving backward, the player F takes this into account in comparing the continuation payoffs from F_4^D and F_4^I. If M_2^D then F's continuation payoff from playing F_4^D is $v[(1-R)r^F + r^M] = 0$, whereas from playing F_4^I is $w(1-R)r^F + zr^M = \frac{9}{2}$. Therefore, F_4 is now history-independent - regardless of M's preceding move, M_2, F will uniquely play F_4^I in order to ensure the D levels for the rest of the stage game. This proves that in this case there exists no Disciplined SPNE as there will never be F_4^D on the equilibrium path.

In order to prove that there exists an Indisciplined SPNE we need to show that in M_2 the level D is not a unique play regardless of the level played in F_2 (for M_1 this is automatically satisfied since the move is simultaneous and $r^M > r^F$). Therefore, move backwards and assume F_2^I. Then, using the above information, M's continuation payoff from playing M_2^D is $b(1-R)r^F + ar^F + b(1-R)r^F + dr^M = -3$ whereas from playing

M_2^I is $2dr^M = 0$. Comparing these two implies that M_2^* will be the best response to F_2 and hence an Indisciplined SPNE exists. □

Appendix F. Proof of Remark 2

Proof. Note that $R = 0$ implies $T(r^M, r_j^F) = r^M$ and therefore $N^M = 1$. Solving backwards, as in Section 4 we know that $F_{n>1}^{j*} \in b(M_1), \forall j$ and $F_1^{j*} \in b(M_1), \forall j$. This implies that all $j \in I$ will select the same moves in all their n_j^F, ie $F_n^j = F_n, \forall n, j$.[29] But due to their differing degrees of commitment they will do so at different points in time. For M to surely-win it is therefore required that $b(F_1^I) = \{M_1^D\}$, which yields the following condition analogous to (6)

$$b\sum_{j=1}^{J} w_j r_j^F + a\sum_{j=1}^{J}(r^M - w_j r_j^F) > dr^M. \tag{25}$$

Rearranging and substituting in the specific payoffs yields (12). □

[29] This will not necessarily be the case under $R > 0$ but the conclusions will be unchanged.