Monetary and Fiscal Policy Interaction With Various Degrees and Types of Commitment [1]

Jan Libich[2]
La Trobe University, Dep. of Economics and Finance, and CAMA

Andrew Hughes Hallett
George Mason University, School of Public Policy, and CEPR

Petr Stehlík
University of West Bohemia, Dep. of Mathematics

Abstract

Monetary and fiscal policies interact in many ways. Recently, the stance of fiscal policy in a number of countries (including the US and EU) has raised concerns about risks for the outcomes of monetary policy. Our paper first shows that these concerns are justified since - under an 'ambitious' fiscal policymaker - inflation bias and lack of monetary policy credibility may obtain in equilibrium even if the central banker is fully independent, patient, and 'responsible'. To reach a possible solution the paper proposes a novel asynchronous game theoretic framework that generalizes the standard commitment concept; most importantly it allows for varying *degrees of commitment*. It is demonstrated that the undesirable scenario can be prevented if monetary commitment is *sufficiently* strong *relative* to fiscal commitment. Interestingly, such strong monetary commitment can not only resist fiscal pressure but also 'discipline' an ambitious fiscal policymaker and achieve socially desirable outcomes for *both* policies (ie also reduce the size of deficit and debt). Furthermore, since it is often the fiscal policymaker that can impose monetary commitment (eg legislate a numerical inflation target), this offers an avenue for governments to tie their hands indirectly if direct fiscal reform seems politically infeasible. We also extend the setting to the European monetary union case with many heterogeneous fiscal policymakers and show that all these findings carry over. We conclude by showing that all our predictions are empirically supported.

Keywords: commitment, asynchronous/alternating moves, monetary vs fiscal policy interaction, Game of chicken, Battle of sexes, inflation targeting

JEL classification: E61, E63, C73

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[2]Corresponding author: La Trobe University, School of Business, Melbourne, Victoria, 3086, Australia. Phone: +61 3 94792754, Email: j.libich@latrobe.edu.au.

1. Introduction

Consider the following situation. One political party makes the claim (call it C for 'claimant') that interest rates and inflation would be significantly higher under the rival party (call it D for 'defendant'). The D party argues this to be misplaced since the country has a fully independent central bank that 'responsibly' targets the natural rate of output. Which party was right? And under what circumstances?[3]

This scenario - to which we will refer throughout as the 'campaign' - highlights the importance of understanding the interaction of fiscal and monetary policy on outcomes of *both* policies. The idea that these policies might interact goes back to Tinbergen (1954), Mundell (1962) and Cooper (1969) but up until recently the models used for policy design treated each policy in isolation. The subsequent 'interaction' literature has mainly examined the *direct interaction* - the ability of the government (fiscal policymaker) to affect monetary policy outcomes through the appointment of the central banker (Rogoff (1985)), optimal contract with the central banker (Walsh (1995)), or through overriding the central banker (Lohmann (1992)).[4]

The focus of the paper is the *indirect interaction* (see eg Sargent and Wallace (1981), Hughes Hallett and Weymark (2005), Dixit and Lambertini (2003), Persson, Persson and Svensson (2006)) which is more subtle and less well understood. It works through spillovers of economic outcomes – variables such as inflation, output, debt, exchange rate, asset prices, or consumer confidence are all affected by *both* policies and they in turn affect *both* policies.

The recent interest in indirect monetary-fiscal interaction has been driven by two factors. First, most industrial countries have made their central banks independent which commonly prevents the direct channel from playing a major role. Second and more important, the stance of fiscal policy in a number of countries has raised understandable concerns about the degree of discipline and commitment in fiscal policies, and about the risks which that may pose in terms of undermining the credibility and the focus of monetary policy. This relates to, among others, the United States and the Euro area.[5]

Our main contribution here is to show the policy interaction in a dynamic game theoretic setting where policymakers may have different degrees of commitment to

[3]Many real world examples of such a situation can be found – eg the 2004 Australian federal election campaign. For the sake of argument assume that the bank cannot be overridden and abstract from any open economy considerations.

[4]For an alterative analysis which encompasses all three of these approaches and explicitly connects them to the reputation literature initiated by Backus and Driffill see Hughes Hallett and Libich (2007).

[5]To demonstrate, since the arrival of the Euro in 1999, the Stability Pact's 3% limit on fiscal deficits has been breached by 6 out of 12 Eurozone members and the Pact itself set aside following a decision in the European Court of Justice. The less often quoted debt limit (at 60% of GDP) was breached by 9 of 12 members in 1999; and 6 of them still breach it in 2007.

their particular regimes and policies. Specifically, we develop a novel asynchronous game theoretic framework that generalizes alternating move games of Maskin and Tirole (1988) and Lagunoff and Matsui (1997).[6] This framework features a combination of simultaneous and sequential moves (perfect and imperfect information) and allows actions to differ in frequency. That enables us to postulate a new concept of commitment that has several advantages over the standard concept - it is more general and more flexible, and hence in many cases more realistic. Most importantly, it allows for: (i) concurrent commitment of more than one player/policy, (ii) partial commitment, and (iii) endogenously determined (optimally selected) commitment.

As a matter of experimental control the rest of our setting is standard. There are two *independent* policymakers in the game – monetary, M, and fiscal, F. Player M sets the level of inflation whereas F chooses the growth rate of nominal debt (size of the budget deficit) and both of these instruments can boost output. Following the literature, both policymakers care about the stability of inflation and the output gap around the chosen target values.

Types of Policymaker/Commitment. The only aspect in which M and F may differ is their target value for the output gap, x_T, as in the literature building on Barro and Gordon (1983). Based on the x_T level we will distinguish two types of policymakers. We refer to those with the socially optimal $x_T = 0$ (who target the natural rate) as *responsible* and those with $x_T > 0$ as *ambitious*. The players have however complete information about their opponent's type (x_T is common knowledge) - this is to separate the effect of uncertainty from the effect of our generalized commitment.

The type of policymaker determines the type of commitment. If a responsible type commits his commitment is called *responsible* whereas if an ambitious type commits such commitment will be referred to as *ambitious*. In Section 3.3, these commitment types will be related to real world arrangements such as explicit inflation targeting, policy transparency, balanced budget rule, unsustainable welfare/health/old-pension schemes etc.

Stage Game Scenarios and Outcomes. To better communicate the intuition we restrict the action space implied by our simple macroeconomic model to two choices in which the policymakers are either 'disciplined', D (ie deliver the socially optimal levels) or 'indisciplined', I (deliver suboptimal levels). In terms of F this expresses having a stable vs growing nominal debt (balanced budget vs deficit). In terms of M this can be interpreted as low vs high inflation (or monetizing the debt).

[6]This existing game theoretic work provides a strong justification and motivation for our general approach; for example, Cho and Matsui (2005) argue that: *'[a]lthough the alternating move games capture the essence of asynchronous decision making, we need to investigate a more general form of such processes. . . '*.

The focus of the paper is on the setting of the above 'campaign' which rececived most attention in the literature; M is responsible, $x_T^M = 0$, but F is ambitious, $x_T^F > 0$ (which can be interpreted as the D type of government).[7] We first show that our simple macroeconomic model can produce a number of stage game outcomes with either a unique or multiple Nash Equilibria, depending on the policymakers' weights on objectives and the structure of the economy.

This step can be interpreted as showing the exact circumstances under which each party in our 'campaign' was right. Importantly, despite M's responsibility, patience, and independence, inefficient M policy outcomes (namely inflation bias and lack of credibility) may still obtain in equilibrium as claimed by the C party. This serves to motivate our analysis and highlights the importance of understanding the MF interaction on outcomes of *both* policies.

Our main (but not exclusive) focus will be on one possible case, the '*Battle scenario*' which has a structure of the Battle of Sexes game and features two pure strategy Nash equilibria, (MD, FD) and (MI, FI), each of them preferred by one player (Figure 1 gives a typical example).

		F	
		FD	*FI*
M	*MD*	1, 0	-½, -½
	MI	-1, -1	0, 1

FIGURE 1. The BATTLE scenario: an example

There are three reasons for this choice. First, MF interactions have sometimes been studied as the 'Game of Chicken' (eg Barnett (2001) or Bhattacharya and Haslag (1999)) and the *Battle* scenario is similar in that it also features two pure and one mixed strategy Nash equilibria.[8] Second, it is the most interesting scenario from the game theoretic point of view as there are equilibrium selection problems - into which our framework provides some novel insights. Third, the results derived in this scenario will imply analogous results in all other scenarios - which we discuss in detail in Section 7.

Standard Commitment: One Degree, One Player. The standard game theoretic concept of commitment involves Stackelberg leadership, ie the first move. This means that only one player can be committed at a time. Further, it is impossible to study partial commitment (a certain degree of it). Introducing this standard

[7] Nevertheless, Section 7 reports results for the situations of a responsible F policymaker, $x_T^F = 0$, and/or ambitious M policymaker, $x_T^M > 0$.

[8] It will be apparent from our macro model that the Battle of Sexes structure better expresses the nature of the current policy interaction with $x_T^M = 0$.

commitment in the *Battle* scenario uniquely selects one of the pure Nash equilibria whereby the first move (leadership) is an advantage. Under F commitment F's preferred outcome (MI, FI) results (in line with the C party's claim); whereas under M commitment M's preferred and socially optimal outcome (MD, FD) will be selected (which is consistent with the D party's defence)

Our Generalized Commitment: Various Degrees, More Players. We introduce the idea of asynchronous games as a way to overcome the restrictions of the standard repeated game. The general setup in discrete and continuous time can be summarized by one parameter, $\theta_t^i = [0, 1]$, which denotes *'the probability that player i's action cannot be altered in time t'*. This nests both main specification of infrequent (staggered) actions in the macroeconomic literature: the Taylor (1979) deterministic and the Calvo (1983) probabilistic schemes.[9] In this paper we focus on *discrete* time with deterministic moves in which the MF interaction has been most often studied. Extensions are examined in Libich and Stehlik (2007a,b).

Let us define player i's **deterministic commitment**, $r^i \in \mathbb{N}$, as *'the number of periods for which player i's action cannot be altered'* (see Figure 2 for an example).

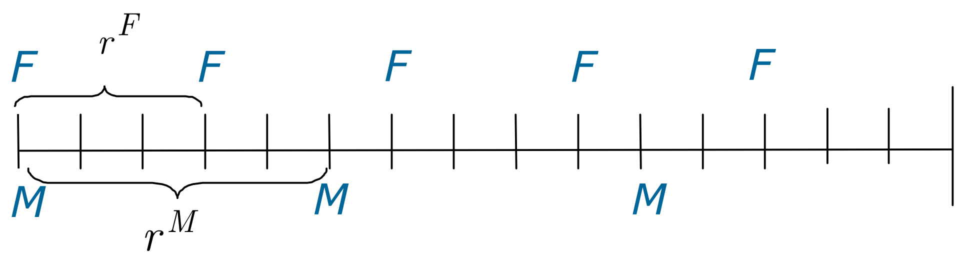

FIGURE 2. An asynchronous game with deterministic commitment - an example of timing of moves with $r^F = 3$ and $r^M = 5$.

To compare the results to the standard repeated game we adopt all its main assumptions; the game starts with a simultaneous move and all past periods' actions are observable (ie games of 'perfect monitoring'). Our game thus combines perfect and imperfect information which is arguably a good description of the real world MF interaction. The deterministic framework further captures the fact that '*Some decisions by economic agents are reconsidered daily or hourly, while others are reviewed at intervals of a year or longer...'* (Tobin (1982) quoted in Reis (2006)) and follows Tobin's call: ...*'It would be desirable in principle to allow for differences among variables in frequencies of change and even to make these frequencies endogenous...'*.

[9]Furthermore, it also encapsulates the standard repeated game (in which $\theta_t^i = 0$, $\forall i, t$) as well as the alternating move game (in which the respective probabilities for the two players i and j are, $\forall t$, $\theta_t^i = \frac{(-1)^t+1}{2}$ and $\theta_t^j = \frac{(-1)^{t+1}+1}{2}$).

Findings. There are two implications of our analysis that are in line with conventional wisdom. First we show that, from the perspective of society, the responsible types of both M and F commitment are desirable whereas the ambitious ones are undesirable. Second, it is shown that, from the perspective of the players, commitment (of any type) is an advantage.

Our main contribution lies in broadening and refining these statements by allowing for various degrees of commitment. We first show that in order for a player's preferred outcome to *uniquely* obtain, his *relative* commitment has to be sufficiently strong. Specifically, it has to be above a certain threshold, $r^i > \overline{r^i}$, that is a increasing function of the opponent's commitment r^j and other variables such as the players' discount factors, their weights on objectives, and the structure of the economy.

Interestingly, it is demonstrated that, under some circumstances, (i) the required degree of relative commitment is arbitrarily low, $r^i > \overline{r^i} = r^j$. In contrast, under some circumstances, namely a very impatient policymaker, (ii) even an infinitely strong commitment is insufficient. Furthermore, under some circumstances, (iii) even if $r^i > r^j$ the outcome preferred by j is more likely in equilibrium than that preferred by i. It should be noted that the latter two findings qualify the intuition of the standard commitment concept. This is because standard commitment always ensures the preferred outcome of the (more) committed player in the Battle of Sexes. That is not always the case under generalized commitment.

Policy Implications. These findings imply mixed news regarding the outcomes of the policy interaction. The bad news - in line with the C party's claim in the 'campaign' - is that *undesirable* M policy outcomes may obtain even if the central bank is independent, responsible, patient, and committed. Hence these central bank characteristics are not sufficient conditions for low inflation and policy credibility.

The good news is that this situation can be avoided if the central bank is *sufficiently strongly* committed relative to F policy. Furthermore, this can indirectly discipline the ambitious F policymaker and achieve socially optimal outcomes for *both* policies. Formally, D uniquely obtains for both policies on the equilibrium path of any *subgame perfect Nash equilibrium.* Intuitively, if the inflation target is sufficiently explicitly stated in the legislation, this creates incentives for F to run balanced budgets since there is no chance of M policy accommodating a deficit. Since it is often F that can impose M commitment (eg legislate the inflation target), doing so may help governments justify or gain political support for a necessary fiscal reform.

Testable Hypotheses. Our analysis has several testable implications: 1) The level of inflation is weakly decreasing in the degree of M commitment (the explicitness/transparency/accountability of the inflation target) as well as in M's patience (degree of central bank goal independence).

Further and interestingly, 2) an explicit inflation target is a substitute for central bank goal independence in achieving time-consistency and credibility. This substitutability offers an explanation for the fact that inflation targets have been made more explicit in countries that had lacked central bank independence in the past such as New Zealand, Canada, UK, and Australia, rather than those with an independent central bank such as the US, Germany or Switzerland.

Finally, 3) M commitment (explicit inflation target) can reduce the size of the budget deficit and debt. This is consistent with the observed fact that despite no major changes to the institutional design of F policy over the past two decades (in contrast to M policy), the outcomes of F policy have improved in many countries, and the greatest improvements (debt reductions) were achieved by explicit inflation targeters.[10]

Section 9 first discusses suitable proxies of these variables and then shows that these predictions are supported by the data (and in doing so it reconciles some conflicting empirical findings of the existing literature). This is supplemented by a case study of Section 9.1.1 kindly written by Don Brash, Governor of the Reserve Bank of New Zealand during 1988-2002. His contribution describes the developments in New Zealand shortly after the adoption of explicit inflation targeting, and highlights the 'disciplining' effect of this M policy arrangement on F policy.

The rest of the paper proceeds as follows. Section 2 presents the macroeconomic model. Section 3 introduces our asynchronous game theoretic framework with generalized commitment. Sections 4 solves the macro model, provides its game theoretic representation, and reports the stage game outcomes. Sections 5 and 6 focus on the *Battle* scenario: the first considers standard commitment whereas the second studies the effect of various degrees of commitment - under both patient and impatient policymakers. Section 7 examines all other scenarios and types of commitment - using the results previously derived. Section 8 extends the analysis to the (European) case with heterogeneous fiscal policy. Section 9 presents related empirical support. Section 10 discusses the robustness of the results and Section 11 summarizes and concludes.

2. Macro Model

The economy can be described by a Lucas supply curve

$$x_t = \mu(\pi_t - \pi_t^e) + \rho g_t, \tag{1}$$

[10] Countries have generally not made their budgetary processes more transparent. For example, countries have typically not legislated a long-term balanced budged rule - which is the F analog of a long-run inflation target in M policy.

where x, π, π^e, and g denote the output gap, inflation, inflation expected by the public, and the growth rate of real debt respectively.[11] The parameters μ and ρ are positive. The growth rate of real debt is then defined as

$$g_t = G_t - \pi_t. \tag{2}$$

where G is the growth rate of nominal debt. The policymakers' discount factors are δ_M and δ_F and their one period utility function is standard:

$$u_t^i = -\beta^i (x_t - x_T^i)^2 - \pi_t^2, \tag{3}$$

where $i \in \{M, F\}$, x_T denotes the output gap target (the inflation target π_T has been normalized to zero), and the parameter $\beta > 0$ expresses relative weight on objectives (the degree of conservatism in Rogoff (1985) terminology).[12]

2.1. **Assumptions.** In line with the literature we assume the socially optimal output gap target to be zero, $x_T^* = 0$. In Sections 5 and 6 we examine the scenario of the 'campaign' in which M is responsible, $x_T^M = 0$, and F is ambitious, $x_T^F > 0$, whereas Section 7 examines the alternatives. M and F are also assumed to have perfect and independent control over their instruments, π and G respectively.[13]

The Public. The private sector agents are assumed to have complete information and rational expectations. As is standard in the literature, expectations can be costlessly adjusted every period. This means that there are no reputation issues and the public's behaviour will not play a major role in the analysis.[14] While there could be an inflation surprise in theory (since M and the public move simultaneously), there will never be a surprise in practice (in equilibrium) as $(\pi_t^e)^* = \pi_t, \forall t$.

Credibility. Following the literature, the term credibility (of the inflation target) will express whether/how inflation expectations deviate from the inflation target.[15]

[11] Our results are robust to the specification of the supply function. For example, they obtain if real debt growth is replaced by nominal debt growth (with a realistic modification of the players' preferences).

[12] In terms of $\beta^M > 0$ it has been forcefully argued that even central banks with a legal 'unitary' or 'hierarchical' mandate (in which price stability is the sole or primary goal) attempt to stabilize output in practice, see eg Cecchetti and Ehrmann (1999) or Kuttner (2004).

[13] Assuming M to be fully independent is a matter of experimental control; we need to separate the indirect effect of MF interaction from the direct dependence effect. Our companion paper Hughes Hallett and Libich (2007) explicitly incorporates various degrees of central bank goal-independence.

[14] For the opposite cases in which (i) reputations matter or (ii) the public's actions also feature some rigidity/commitment (due to costly wage bargaining and/or information processing) see Hughes Hallett and Libich (2007) and Libich and Stehlik (2007a) respectively.

[15] Specifically, we follow the interpretation of Faust and Svensson (2001) who quantify credibility as $C_t = -|\pi_T - \pi_t^e|$ (which in our equilibrium ends up being equivalent to $C_t = -|\pi_t|$). If $C_t = 0$ then we will call the inflation target to be credible, whereas if $C_t < 0$ the target and monetary policy will lack credibility. Also note that our setting allows us to make the distinction between *credibility of policy* (target) and *credibility of regime.* The latter can be modelled as

Long-run Perspective. Since our interest lies in the effect of commitment on policy outcomes we have adopted a long-run perspective and focus on average/trend outcomes of the game. Therefore, the economy in (1) is deterministic - it does not feature shocks. This implies that the policymakers' instruments should also be interpreted as *average/trend/long-run levels*.

3. Generalized Commitment

Since we model the interactions between two players, each of whom has one instrument at his/her disposal, we will introduce the framework for this special case. For a more general setting see Libich and Stehlik (2007a,b).

Denote the probability that player $i \in \{M, F\}$ cannot move in period t by θ_t^i. Then the general discrete or continuous case can be summarized as follows

$$0 \leq \theta_t^i \leq 1. \tag{4}$$

In the discrete framework time is denoted by $t \in \mathbb{N}$. (4) then nests the standard repeated game as well as the alternating move game (see footnote 9). One natural subcase is the popular scheme of Calvo (1983) in which $\theta_t^i = \theta^i, \forall i, t$ and $\theta^i \in (0, 1)$ - whereby θ^i can be interpreted as *probabilistic commitment*. As this is examined in Libich and Stehlik (2007b), we focus on the *deterministic* specification of Taylor (1979) here, in which

$$\theta_t^i = \begin{cases} 0 & \forall i \text{ and } \forall t = 1 + (n-1)r^i \text{ where } n, r \in \mathbb{N}, \\ 1 & \text{otherwise.} \end{cases} \tag{5}$$

Not only is this case more intuitive and easier to analyze than the Calvo setting (avoiding the unrealistic possibility that an 'unlucky' player may never be able to move) but it is representative of asynchronous decision making. It will be discussed later that the results under the Calvo specification are analogous.

3.1. Assumptions.

We adopt all the assumptions of a standard repeated game - a number of alternative specifications are discussed in Section 10. First, both the type and degree of commitment are constant throughout each game. Second, they are common knowledge. Third, all past periods' moves can be observed. Fourth, the game starts with a simultaneous move. Fifth, players are rational, have common knowledge of rationality and for expositional clarity they have complete information about the structure of the game and opponents' payoffs.

In addition to our definition of deterministic commitment r^i in Section 1, we now have the following.

Definition 1. *An* ***unrepeated asynchronous game with deterministic commitment*** *is an extensive game that starts with a simultaneous move, continues*

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the deviation of the public's (or opponent's) perception of r^i from the actual r^i. Nevertheless, throughout this paper we assume the regime to be fully credible, ie all players will know their opponents' actual r^i.

with 'committed' moves every r^i *periods, and finishes after* T *periods, where* $T \in \mathbb{N}$ *denotes the 'least common multiple' of* $r^i, \forall i$.

An example of such game in the form of a time line is presented in Figure 2 in which $T(r^M = 5, r^F = 3) = 15$.

3.2. **(Non)-Repetition.** While this asynchronous game can be repeated we will restrict our attention to the unrepeated game (as depicted in Figures 2-3). This is possible because we will be deriving conditions under which the *unique efficient* outcome *uniquely* obtains on the equilibrium path of the unrepeated game. Due to these two uniqueness properties, if these conditions are satisfied repeating the game and allowing for reputation building of some form would not affect the derived equilibrium.[16] The uniqueness also implies that we can only focus on pure strategies without loss of generality.

3.3. **Interpretation.** By definition the degree of commitment, r^i, relates to the inability to change the policy course/actions. The main reason for such inability is the fact that some policies may be *legislated* and hence the policymaker cannot alter them readily at will. Therefore, r^i can be interpreted as the degree of *explicitness* with which the objectives/targets of the respective policies are stated in the legislation.[17] The underlying assumption is that, the more explicitly/transparently a certain policy goal is grounded in the legislation, the less frequently it can be altered (in the Taylor (1979) deterministic sense) and the less likely it is to be altered (in the Calvo (1983) probabilistic sense).

In order to give specific interpretations of r^i we need to factor in two things. First, due to our long-run perspective, the policy instruments represent setting average/trend levels. Second, as indicated in Section 1, the effect of a degree of commitment r^i depends on the type of commitment, x_T^i.

In terms of a responsible M commitment, since π expresses choosing some average inflation - or equivalently, a certain long-run inflation target - r^M can be interpreted as the degree of explicitness with which the socially optimal inflation target is legislated. As a real world example of a deterministic r^M, the 1989 Reserve Bank of New Zealand Act states that the inflation target may only be changed in a Policy Target Agreement between the Minister of Finance and the Governor, and that this can only be done on *pre-specified regular* occasions (eg when a new Governor is appointed).[18]

[16]In this sense we can think of our analysis as the worst case scenario in which reputation cannot help in cooperation.

[17]Our interpretation is in line with Geraats (2002) and Eijffinger and Geraats (2006) and their concept of 'political transparency' which has three elements, namely 'formal objectives', 'quantitative targets', and 'institutional arrangements'. All three are officially grounded in the legal framework of the policy.

[18]Since late 1990 the PTA was 'renegotiated' five times, ie roughly every three years. Only on two occasions the target level was changed: in 1996 from 0-2% to 0-3% and in 2002 to 1-3%.

It should further be noted that the absence of a legislated numerical target may not necessarily imply $r^M = 1$; it has been argued that many countries pursue an inflation target implicitly (including the US, Goodfriend (2003); or the Bundesbank and the Swiss National Bank in the 1980-90s, see Bernanke, et al. (1999)). In such cases we have $r^M > 1$.[19]

In terms of an ambitious F commitment, $x_T^F > 0$, this can be interpreted as an *unsustainable* setting of public expenditures, or welfare/health/pension schemes. This is because such schemes are legislated and therefore pre-commit F policy to certain actions in the future. Such schemes make it impossible for G to be genuinely chosen or altered every period.[20]

3.4. **Notation.** Denoting n^i to be the i's player's n'th move, and N^i the number of moves in the unrepeated game, it follows that $N^i = \frac{T(r^M,r^F)}{r^i}$. Also, M_n^l and F_n^l will denote a certain action $l \in \{D, I\}$ at a certain node n^i; eg F_2^I refers to F's indiscipline in its second move. Assume $r^i > r^j$ and denote $\frac{r^i}{r^j} \geq 1$ to be the players' relative commitment where $i \in \{M, F\} \ni j$. Further, $\lfloor \frac{r^i}{r^j} \rfloor \in \mathbb{N}$ will be the integer value of relative commitment (the floor) and $R = \frac{r^i}{r^j} - \lfloor \frac{r^i}{r^j} \rfloor = [0, 1)$ denotes the fractional value of relative commitment (the remainder).[21] Further, we denote $b(.)$ to be the best response. For example, $F_1^D \in b(M_1^D)$ expresses that F^D is F's best response to M's initial D move and $b(M_1^D) = \{F_1^D\}$ expresses that it is the unique best response. Recall that a star denotes optimal play. Thus $F_1^* \in b(M_1)$ expresses that F's optimal play in move 1 is the best response to M's first move. Finally, various threshold levels will be denoted by upper or lower bar. For example, $\overline{r^M}$ will be a *sufficient* M commitment level (that obtains for all R) whereas $\overline{r^M}(R)$ will be a *necessary and sufficient* M commitment level that is a function of R.

3.5. **Recursive Scheme.** Throughout the proofs we will be taking advantage of the recursive scheme implied by the setup. Again assuming $r^i > r^j$, let k_n denote the number of periods between the n^i-th move of player i and the immediately following move of player j (for an example see Figure 3).

From this it follows that the number of periods between the $(n^i + 1)$-th move of i and the immediately preceding move of j equals $r^i - k_{n+1}$. Using these facts, we

[19] It is also important to realize that due to the long-run perspective r^M does *not* express how frequently the short-run interest rate instrument of M policy can be adjusted. This means that, in a stochastic environment, shocks can still be stabilized since the tager must be achieved *on average* (over the long-run/business cycle): see Libich (2006) for a thorough analysis of the relationship between M's commitment and short-run flexibility to stabilize inflation and output in the presence of disturbances.

[20] For completeness, a responsible F commitment, $x_T^F = 0$, would be interpreted analogously to $x_T^M = 0$ as the explicitness with which a long-run balanced budget rule is legislated.

[21] It will be evident that R plays an important role as since it defines the exact type of asynchronicity in the game.

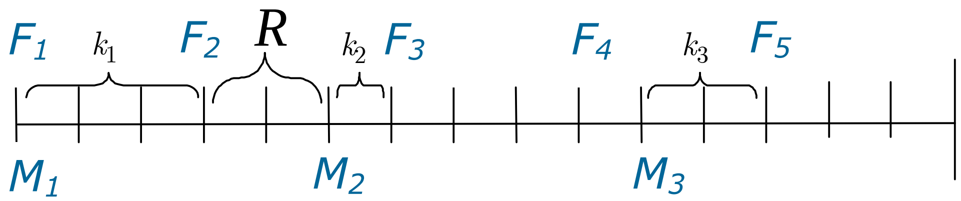

FIGURE 3. Unrepeated asynchronous game with deterministic commitment: illustration of the recursive scheme and of R, k and n^i.

can summarize the recursive scheme of the game as follows:

$$k_{n+1} = \begin{cases} k_n - Rr^j & \text{if} \quad k_n \geq Rr^j, \\ k_n + (1-R)r^j & \text{if} \quad k_n < Rr^j, \end{cases} \tag{6}$$

Generally, k_n is a non-monotone sequence.

3.6. **History and Future.** By convention, history in period t, h_t, is the sequence of actions selected prior to period t and future in period t is the sequence of current and future actions. It follows from our perfect monitoring assumption that h_t is common knowledge at t. Let us refer to moves in which a certain action $l \in \{D, I\}$ is selected for all possible histories as 'history-independent'.

3.7. **Strategies and Equilibria.** A strategy of player i is a vector that, $\forall h_t$, specifies the player's play $\forall n^i$. The asynchronous game will commonly have multiple Nash equilibria. To select among these we will use a standard equilibrium refinement, subgame perfection, that eliminates non-credible threats. Subgame perfect Nash equilibrium (SPNE) is a strategy vector (one strategy for each player) that forms a Nash equilibrium after any history h_t.[22]

Given the large number of nodes in the game reporting fully characterized SPNE would be cumbersome. We will therefore focus on the *equilibrium path* of the SPNE, ie actions that actually get played.[23] In doing so we will use the following terminology regarding two symmetric types of SPNE we are interested in.

Definition 2. *Any SPNE that has, on its equilibrium path, both policymakers playing D in all their moves,* $(i_n^D)^*, \forall n, i$, *will be called* ***Disciplined****. Such outcome will be referred to as* ***General Discipline****. Any SPNE that has, on its equilibrium path, both policymakers playing I in all their moves,* $(i_n^I)^*, \forall n, i$, *will be called* ***Indisciplined****. Such outcome will be referred to as* ***General Indiscipline****.*

[22]Note that the specification of the players' utility implies that all our SPNE will also be Markov perfect equilibria: see Maskin and Tirole (2001).

[23]To demonstrate, for the example in Figure 2 each SPNE consists of $\sum_{s=1}^{r^F} \sum_{f=1}^{r^M} 2^{(s+f-1)} = 254$ actions whereas on its equilibrium paths there are $r^F + r^M = 8$ actions.

3.8. **Discounting.** To make the exposition simpler we will first examine the game under the assumption of (fully) *patient* policymakers, $\delta_i = 1, \forall i$, and then consider the effect of the policymakers' *impatience,* $\delta_i < 1$. As the intuition of commitment is independent of the players' discount factor, most of the results will carry over. It will be shown that policymakers' impatience can, depending on the circumstances, either improve or worsen the degree of cooperation and associated outcomes.

4. Solution of the Model

Focusing on the stage game we have, using (1)-(3), the following reaction functions under rational expectations

$$\pi_t^* = \frac{\beta^M(\rho-\mu)(\rho G_t - x_T^M)}{1+\beta^M\rho(\rho-\mu)} \text{ and } G_t^* = \pi_t + \frac{x_T^F}{\rho}. \tag{7}$$

Substituting one into the other we get the following equilibrium outcomes

$$\pi_t^* = \beta^M(\rho-\mu)(x_T^F - x_T^M) \text{ and } G_t^* = \frac{x_T^F}{\rho} + \beta^M(\rho-\mu)(x_T^F - x_T^M). \tag{8}$$

Several results are implied for our 'campaign' setting; we depict only those related to M policy (for a number of additional results see Hughes Hallett, Libich and Stehlik (2007a,b) that relate these to the recent experiences of eg Japan and Argentina).

Proposition 1. *In the standard one-shot game without commitment and with a responsible M and an ambitious F, $x_T^M = 0 < x_T^F$, the following claims hold:*
(i) The setting of M policy is not independent of F policy (for all $\rho \neq \mu$).
(ii) For almost all parameter values the inflation target is time-inconsistent and lacks credibility, whereby both inflation and deflation bias can occur.
(iii) A more conservative and more responsible central banker may increase the inflation bias and reduce credibility.

Proof. See Appendix A (and Figure 4 that demonstrate claims the first two claims graphically) □

Proposition 1 serves to motivate our analysis by showing that the concerns about the impact of F policy on M policy outcomes have theoretical foundations. Among other, it supports the C party's claim that the inflation target may be time-inconsistent and lack credibility despite M's full independence, conservatism, and responsibility. It is interesting to consider why a responsible M may find it optimal to inflate. By doing so, M attempts to decrease the real value of the debt, which would then reduce the expansionary effect of F policy and thus stabilize output closer to potential. Claim (iii) is

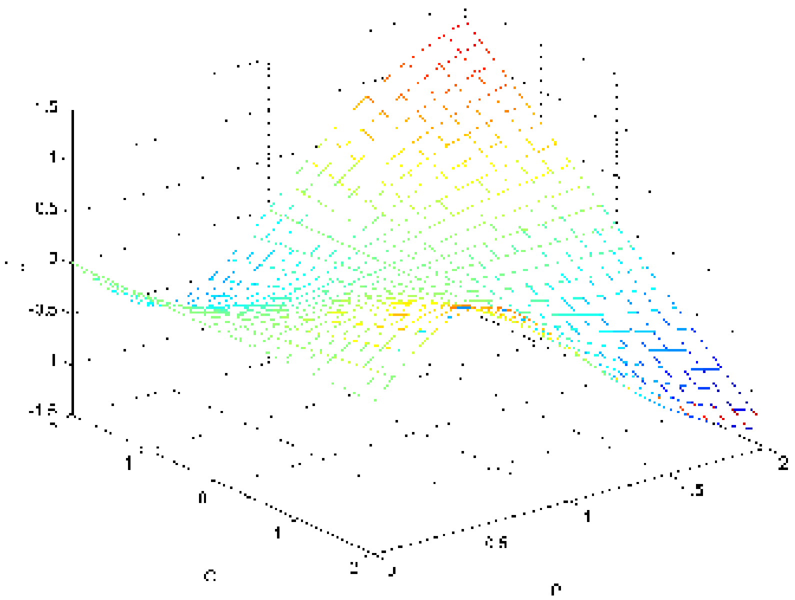

FIGURE 4. The M policy's optimal response as a function of the the size of the nominal debt/deficit G and F policy potency ρ The parameters have been set at $\mu = \beta^M = \beta^F = x_T^F = 1$.

4.1. **Game Theoretic Representation.** Following the gametheoretic litarture, we will for clarity truncate the players' action sets from continuous to only two action levels for each policymaker, D and I. We will choose the two levels of interest (as for example Cho and Matsui (2005)) - the target level and the time-consistent level

$$\pi^D = G^D = 0 \text{ and } \pi^I = \pi^*, G^I = G^*.^{24} \tag{9}$$

The game in its general form is summarized in the payoff matrix of Figure 5 in which $\{a, b, c, d, v, w, y, z\}$ denote payoffs that are functions of the deep parameters of the model.

Definition 3. *The various scenarios of the MF interaction stage game will be defined as follows (all their appropriate Nash equilibria are in brackets):*
1) ***No-gap*** $((MD, FD))$; *2)* ***M-gap*** $((MI, FD))$; *3)* ***F-gap*** $((MD, FI))$; *4)* ***M&F-gap*** $((MI, FI))$; *5)* ***Chicken*** $((MD, FI), (MI, FD))$; *6)* ***Coordination*** $((MD, FD),$

[24]This specification implies that if $\pi^* = 0$ and $G^* = 0$ the D and I levels are identical, ie each player only has one action available. In such situations we will treat the players' play as D. To ensure that the game is meaningful (ie that under all circumstances at least one player has two options) we will impose $x_T^F > 0$ unless otherwise stated.

		F	
		FD	*FI*
M	*MD*	a, v	b, w
	MI	c, y	d, z

FIGURE 5. General payoffs

(MI, FI) *where players prefer the same Nash); and 7)* ***Battle*** $((MD, FD), (MI, FI)$ *where players prefer a different Nash).*

4.2. **Stage Game Outcomes Under Responsible M and Ambitious F.** For most of the paper (in Sections 5-7.2), we will examine this case of interest - that has been the focus of most of the recent literature as well the situation described in the 'campaign'.

Proposition 2. *The MF interaction stage game described by* (1)-(3),(9), *and* $x_T^M = 0$, *can have all the scenarios listed in Definition* (3) *except M-gap and Chicken.*

Proof. See Appendix B. □

Proposition 2 complements Proposition 1 in showing that socially undesirable outcomes may obtain, for both policies, even if M is fully independent and targets the natural rate. Out of all the possible scenarios Sections 5 and 6 examine (for the reasons explained in the introduction) the most interesting *Battle* scenario. Sections 7.1 and 7.2 then discuss the remaining scenarios.

5. THE BATTLE SCENARIO WITH STANDARD COMMITMENT

Equation (8) in combination with (1)-(3) and $x_T^M = 0 < x_T^F$ implies that the *Battle* scenario in its *general* form can be summarized as follows

$$a > c, a > d > b \text{ and } z > y, z > v > w. \tag{10}$$

For illustration we will also use a *specific Battle* game as reported in Figure 1

$$a = z = 1 > d = v = 0 > b = w = -\frac{1}{2} > c = y = -1. \tag{11}$$

As there are two pure strategy Nash equilibria in this case, each one preferred by a different player, neither is more likely to be selected (the focal point argument cannot be used). Therefore, the Nash in *mixed strategies*, which yields inferior payoffs to both players, is a possibility and reason for concern. The commonly used solution to this problem is to consider players' commitment.

The standard commitment concept imposes Stackelberg leadership of one player which turns out to be an advantage in this game. Under M's commitment (leadership), player M's preferred outcome (MD, FD) obtains in equilibrium whereas under F commitment F's preferred outcome (MI, FI) results. In order to examine the robustness of these conclusions our generalized framework examines various degrees of commitment.

6. The Battle Scenario with Generalized Commitment

Our goal is to study how the macroeconomic outcomes of the policy interaction may vary with various degrees of M and F commitment. This will, among other, identify the circumstances under which each party's 'campaign' claim obtains. The next two subsections focus on situations when the socially optimal disciplined outcomes are *surely* achieved despite F's ambitions (in line with the D party's defence) whereas the third one analyzes those under which this is not the case (in line with the C party's claim).

6.1. **Patient Policymakers.** For illustration purposes we first examine the game without players discounting the future. Furthermore, we support the results of the *general* game, where only (10) is required to hold, with those of the *specific* game in (11).

Proposition 3. *Consider the Battle scenario in which* (10) *holds and* $\delta_F = \delta_M = 1$. *The sufficient condition,* $\forall R$, *for General Discipline to uniquely obtain, that is for any SPNE of the game to be Disciplined is*

$$r^M > \overline{r^M} = \begin{cases} \left(\frac{a-b}{a-d} + \frac{v-w}{z-y+v-w}\right) r^F & \text{if } \frac{a-b}{a-d} + \frac{v-w}{z-y+v-w} > 2, \\ \left(1 + \frac{v-w}{z-y+v-w}\right) r^F & \text{otherwise.} \end{cases} \tag{12}$$

In the specific Battle game in which (11) *holds this reduces to*

$$r^M > \overline{r^M} = \frac{6}{5} r^F. \tag{13}$$

Proof. To prove the claims it suffices to show that under the stated circumstances π^D is M's unique best play in all his nodes for all histories h_t, ie every optimal move M_n is 'history independent'. As F's unique best response to MD is FD this will then ensure G^D throughout the equilibrium path as well. See Appendix C for the details of the proof. □

Intuitively, the fact that M is never willing to accommodate the deficit and ready to contract the economy if necessary eliminates the incentive of F to run deficits and accumulate debt. We can think of this as some time of punishment by M. Note however that unlike in a standard repeated game (of the Barro and Gordon (1983) type) the punishment is not arbitrary - it is M's optimal play (the cost in terms of output loss are outweighted by the future gain of stable inflation and output) and its length is uniquely determined by the degree of policy commitments. It is also illustrative to consider why some low relative commitment values (in the specific *Battle* game in the interval $\frac{r^M}{r^F} \in [0, \frac{6}{5}]$, fail to uniquely deliver General Discipline. It is because the relative length of M's punishment is insufficient to discourage F from running deficits.

Note that for all types of asynchronicity, R, and all general values of the payoffs satisfying (10), the threshold $\overline{r^M}$ is finite. It therefore follows that, under a fully patient M, a sufficient value of M commitment that uniquely achieves the social optimal outcomes $\pi^D = G^D = 0$, exists for all parameter values. However, in contrast to the standard concept of commitment, our framework gives us additional valuable information. Specifically, it tells us the exact degree of commitment that is required to do so - as a function of various variables. The following Corollary, that follows from inspection of (12), summarizes the various relationships:

Corollary 1. *Consider the Battle scenario in which* (10) *holds. The sufficient degree of M commitment that not only ensures optimal M policy outcomes but also disciplines F policy,* $\overline{r^M}$ *from Proposition 3, is increasing in* r^F, d, v, y *and decreasing in* a, b, w, z.

The payoffs $\{a, b, c, d, v, w, y, z\}$ that determine the threshold value $\overline{r^M}$, and hence the policy outcomes, are functions of the deep parameters of the model. That is, they depend on the players' preferences and the structure of the economy.[25] Note that most parameters affect $\overline{r^M}$ in opposite directions for the two policymakers. For example, M's higher inflation cost (lower d) reduces $\overline{r^M}$ whereas F's higher inflation cost (lower z) increases $\overline{r^M}$.

Relating this back to our 'campaign', if (12) is satisfied then the D party's defence was justified since the outcomes of M policy are not endangered by F policy's 'ambitions'. But if (12) does not hold then the C party's claim was well placed.

6.2. **Impatient Policymakers.** In this section we consider a more general setting in which the policymakers discount the future and show that the qualitative nature of the results is unchanged. Nevertheless, several novel insights emerge that qualify the intuition of the standard commitment concept. To separate the effects of F's and M's discounting we examine each in turn.

6.2.1. *F's Impatience.* This section shows that F's discounting may *weaken* the above sufficient conditions for the uniqueness of General Discipline.

Proposition 4. *Consider the Battle scenario in which* (10) *holds, and assume* $\delta_M = 1$, $0 \leq \delta_F \leq \overline{\delta_F} < 1$ *where* $\overline{\delta_F}$ *is some upper bound, and* $a > 2d - b$. *Then*

[25]A number of literatures have examined how in the real world these depend on underlying factors such as Union power, the way agents form expectations, political economy factors (lobby groups, political cycles), institutional setting of both policies etc.

for General Discipline to uniquely obtain it suffices that

$$r^M > \overline{r^M} = r^F. \tag{14}$$

Proof. We claim that for some parameter values (including those of the specific *Battle* game in (11)) the sufficient threshold is $\overline{r^M} = r^F$ and hence *any* $r^M > r^F$ uniquely ensures the D actions for both policies - for the proof see Appendix D. □

We explicitly formulate this result since it shows that General Discipline can *uniquely* obtain in a game theoretic setting that approaches the standard repeated game. While it only applies under sufficiently impatient F, this is not entirely unrealistic as one would expect F's impatience to go hand in hand with F's 'ambitions' - both are likely to be driven by the same 'political economy factors' (see eg the literature on political business cycles initiated by Nordhaus (1975)).

6.2.2. *M's Impatience.* This section shows that M's impatience strengthens the above sufficient condition, ie it makes it more difficult for General Discipline to uniquely obtain. This is an intuitive result. Perhaps surprisingly, in contrast to both Proposition 3 and to the standard commitment concept, it also shows that if a player is very impatient then even his infinitely strong commitment may be insufficient to uniquely ensure General Discipline. Several policy related findings then follow that offer testable hypotheses.

Proposition 5. *Consider the Battle scenario in which* (10) *holds and some threshold discount factor*

$$\overline{\delta_M} = \sqrt[r^F]{\frac{d-b}{a-b}} \overset{(11)}{=} \sqrt[r^F]{\frac{1}{3}}, \tag{15}$$

(i) If M is sufficiently patient, $\delta_M > \overline{\delta_M}$, then there exists $\overline{r^M} \in \mathbb{N}$ such that, for all $r^M > \overline{r^M}$ and $\forall r^F, R, \delta_F$, General Discipline uniquely obtains.
(ii) If M is insufficiently patient, $\delta_M < \overline{\delta_M}$, then even an infinitely strong M commitment, $r^M \to \infty$, does not uniquely ensure General Discipline.

Proof. The proposition reports the threshold discount factor for both the general game and the specific game (in which also (11) holds and hence the $\overset{(11)}{=}$ notation). Note that (15) yields $0 < \overline{\delta_M} < 1$ for all assumed values, which follows from $a > d > b$ in (10). For the details of the proof see Appendix E. □

Claim (ii) of Proposition 5 stands in stark contrast to the standard commitment concept in which the committed player uniquely ensures his preferred equilibrium in the *Battle* scenario regardless of his impatience, that is $\forall \delta_M$. Relating this to the 'campaign', this result further strengthens the foundations for the C party's claim.

While Proposition 5 reports the sufficient bound $\overline{\delta_M}$, it does not provide the sufficient commitment level $\overline{r^M}$ - it only shows its existence. This is because

Proposition 5 is proven regardless of the value of R and we have seen in the proof of Proposition 3 that the necessary and sufficient commitment level is a function of R, $\overline{r^M}(R)$. Proposition 3 nevertheless showed that while the thresholds $\overline{r^M}(R)$ for $R \in (0,1)$ differ quantitatively from $\overline{r^M}(0)$, they are qualitatively the same, see eg (32) in Appendix C. Therefore we will investigate $\overline{r^M}(0)$ under impatience and then extend our conclusions to the remaining R cases.

Proposition 6. *Consider the Battle scenario in which* (10) *holds and* $R = 0$. *The threshold* $\overline{r^M}(0)$ *is increasing in* r^F *and decreasing in* δ_M, *the latter implying that* M*'s commitment and patience are substitutes in achieving General Discipline.*

Proof. Appendix F shows that the necessary and sufficient M commitment level is

$$r^M > \overline{r^M}(0) = \log_{\delta_M}\left(\frac{a-b}{a-d}\delta_M^{r^F} - \frac{d-b}{a-d}\right) \stackrel{(11)}{=} \log_{\delta_M}\left(\frac{3}{2}\delta_M^{r^F} - \frac{1}{2}\right). \tag{16}$$

from which the implied necessary and sufficient M patience threshold $\overline{\delta_M}(0)$ is equal to the sufficient threshold $\overline{\delta_M}$ from (15). These thresholds are plotted in Figure 6 which demonstrates the claims graphically. For formal proofs see Appendix F. □

Remark 1. *Proposition 6 implies that (i) the existence result of Proposition 5 (as well as other results of Section 6.1) are robust to players' discounting; and that (ii) a less patient M needs to commit more strongly (make its inflation target more explicit) to uniquely ensure the target's credibility.*

We later report empirical evidence for result (ii).

6.3. **Insufficient M Commitment.** To complement Sections 6.1 and 6.2 that focused on the situations of sufficient M commitment, $r^M > \overline{r^M}$, this section briefly examines the outcomes under $r^M \leq \overline{r^M}$. It should now be apparent that all our previous results apply analogously.

Corollary 2. *Consider the Battle scenario in which* (10) *holds. If*

$$r^F > \overline{r^F}, \text{ or equivalently, } r^M < \underline{r^M}, \tag{17}$$

where $\overline{r^F}$ *and* $\underline{r^M}$ *are some 'mirror images' of* $\overline{r^M}$ *derived in Sections 6.1-6.2, then General Indiscipline uniquely obtains.*

Specifically, $\overline{r^F}$ is obtained from $\overline{r^M}$ by swapping all the corresponding variables and payoffs of players M and F. Conversely, $\underline{r^M}$ is some reciprocal of $\overline{r^M}$ that corresponds (and is implied by) $\overline{r^F}$. For example in the specific *Battle* scenario under patient policymakers, General Discipline uniquely obtains if $r^M > \overline{r^M} = \frac{6}{5}r^F$

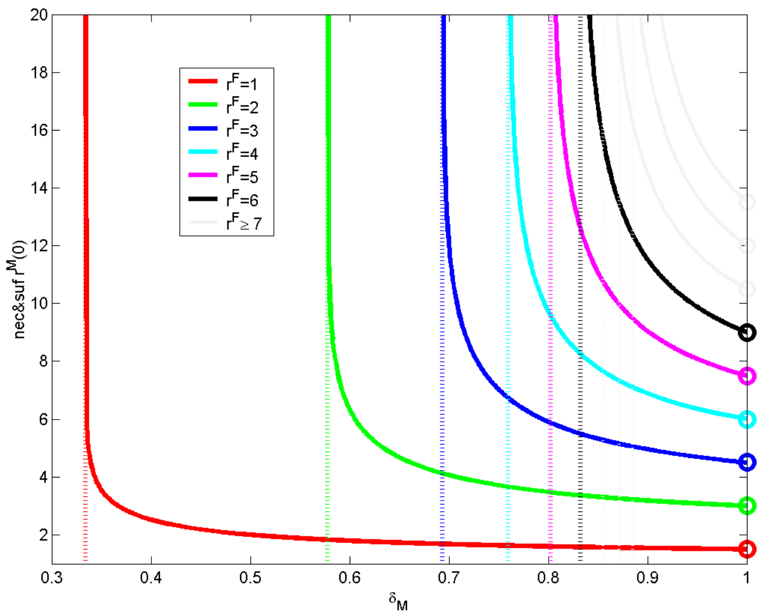

FIGURE 6. The negative relationships of $\overline{r^M}(0)$ with δ_M and r^F (from (16) for the specific game (11)). Dotted asymptotes correspond to bounds $\overline{\delta_M}(0)$ for each particular r^F from (15).

(see (13)) whereas General Indiscipline uniquely obtains if $r^F > \overline{r^F} = \frac{6}{5}r^M$, which is equivalent to $r^M < \underline{r^M} = \frac{5}{6}r^F$.[26]

Recall that $\overline{r^M}$ - and hence $\overline{r^F}$ and $\underline{r^M}$ - only exist if the more committed policymaker is sufficiently patient. Again, the threshold patience levels are mirror images of $\overline{\delta_M}$. For example, the equivalent of threshold $\overline{\delta_M} = \sqrt[r^F]{\frac{d-b}{a-b}} \stackrel{(11)}{=} \sqrt[r^F]{\frac{1}{3}}$ from (15) is $\overline{\delta_F} = \sqrt[r^M]{\frac{v-w}{z-w}} \stackrel{(11)}{=} \sqrt[r^M]{\frac{1}{3}}$.

We can therefore conclude that under $r^M < \underline{r^M}$ (ie $r^F > \overline{r^F}$) the C party's claim will *surely* be realized as General Indiscipline uniquely obtains. Our companion paper Libich and Stehlik (2007b) examines in detail the intermediate region $\underline{r^M} \leq r^M \leq \overline{r^M}$ and shows that the C party's claim *may or may not* be realized. This is because in this interval there are either (i) both Disciplined and Indisciplined SPNE, or (ii) only one of these two types, or (iii) neither of them (in which case all SPNE feature both D and I on the equilibrium path).

The following Corollary implies another testable hypothesis of our analysis.

[26]Since the payoff of the specific game are symmetric, the outcomes remain the same not only qualitatively but also quantitatively.

Corollary 3. *Consider the Battle scenario in which* (10) *holds. If* $r^M < \underline{r^M}$ *(ie* $r^F > \overline{r^F}$*) then* π^D *is time-inconsistent and lacks credibility; and the average levels of both* π *and* G *are higher than under* $r^M > \overline{r^M}$.[27]

Proof. See Appendix G. □

The following result is perhaps surprising as it qualifies the intuition of the standard commitment concept.

Proposition 7. *Consider the Battle scenario in which* (10) *holds and* $r^M > r^F$*. There exist parameter values under which the game has some Indisciplined SPNE but no Disciplined SPNE.*

Proof. See Appendix H. □

Arguably if General Discipline is infeasible whereas General Indiscipline is feasible, it is reasonable to conclude that the latter outcome will be more 'likely'. The fact that this happens under $r^M > r^F$ contrasts the standard commitment solution in which the (more) committed player *always* gets its preferred outcome in the *Battle*. Intuitively, in this example it occurs because M is very averse to output variability (insufficiently conservative) and hence the potential variability cost will discourage him from disinflating. Put differently, lack of conservatism may reduce the potency of M commitment, which further strengthens the claim of the C party.

7. Other Scenarios and Types of Commitment

This section will first consider the remaining scenarios of our case of interest, $x_T^F > 0 = x_T^M$, and then briefly discuss the results under an ambitious M and/or responsible F.

7.1. **The Coordination scenario.** As implied by Proposition 2 this scenario is one of the possible stage game outcomes. It is similar to the *Battle* scenario in that it features two Nash equilibria. But it differs in that one of the Nash is preferred by both players (under $x_T^M = 0$ it is the socially desirable (MD, FD) outcome which is preferred also by F since $v > z$). Thus, while there also exist potential equilibrium selection problems, these are not as pronounced since a focal point argument now applies. Hence we would imagine that the concerns about indisciplined F policy are less pressing. Nevertheless, it is apparent that all the above findings carry over. Specifically if $r^M > \overline{r^M}(R)$ with the threshold derived above, General Discipline uniquely obtains.

[27]Note that for a part of the parameter space with $\underline{r^M} \leq r^M \leq \overline{r^M}$ inflation *variability* is also higher than under $r^M > \overline{r^M}$, for more details see Libich and Stehlik (2007a).

7.2. **The F-gap, M&F-gap, and No-gap Scenarios.** In these scenarios the standard commitment concept does not change the outcome of the game; and the same is true for our generalized commitment. This is because one player (F in the former two, and M in the latter scenario) has a dominant strategy in the stage game.

7.3. **An Ambitious M Policymaker, $x_T^M > 0$.** This setting arguably describes the real world situation in some developing countries. One of its causes may be *direct* involvement of an ambitious government in M policy, ie lack of central bank independence. For that case we can extend the result of Proposition 2.

Proposition 8. *The MF policy stage game described by* (1)-(3)*, and* (9) *can have all the scenarios listed in Definition 3 except Chicken.*

Proof. See Appendix I. □

In comparison to the case with $x_T^M = 0$ one additional scenario, *M-gap,* is possible and its existence under $x_T^M > 0$ is intuitive. What is perhaps surprising is the fact that M's greater ambition as well as F's greater ambition can, under some circumstances, reduce the inflation bias. Formally, from (8) π_t^* is decreasing in either x_T^M or x_T^F.

7.4. **A Responsible F Policymaker, $x_T^F = 0$.** Under $x_T^F = x_T^M = 0$ it follows from (8) that $\pi_t^* = G_t^* = 0$ and hence $g_t^* = x_t^* = 0$. Therefore, the degree of commitment in M and F policy does not affect the policy outcomes if both are the responsible type. Nevertheless, it can be argued that caution should be exercised under incomplete information.

Remark 2. *If there is uncertainty about the value of x_T^F (as it may change over time with eg the political cycle), implementing a sufficiently high M commitment acts as a* ***credible threat*** *to F and as a* ***'credibility insurance'*** *to M.*

The last remaining case $x_T^F = 0 < x_T^M$ is arguably unlikely since M's ambitions in the real world, if any, are believed to be driven by F's ambitions. Nevertheless, if this case was to apply then the above intuition would still carry over. In particular, for the socially optimal outcomes to obtain, the responsible F policymaker would have to be sufficiently strongly committed relative to the ambitious M policymaker.

8. **Heterogeneous Fiscal Policy in a Monetary Union**

An advantage of our game theoretic approach is to be able to elegantly extend and generalize our analysis by incorporating any number of players. To demonstrate, let us examine the case in which F is heterogeneous, ie there are various fiscal policymakers of potentially different economic size (influence/importance) and with differing degrees of commitment. This arguably describes the situation in the European Union with a common currency and hence common M policy but independent F policies.

The players' set is then $I = \{M, F^j\}$ where $j \in [1, J]$ denotes a certain country, r_j^F denotes this country's degree of F commitment, and f_j denotes this country's relative economic size such that $\sum_{j=1}^{J} f_j = 1$. We find it natural to focus on these two types of F heterogeneity keeping the remaining characteristics the same across j's, ie $\forall j$ we have $x_T^{Fj} = x_T^F > 0, \beta^{Fj} = \beta^F$, and $\delta_F^j = \delta_F$. Also, let us depict the simple case with patient players in which $\delta_F = \delta_M = 1$. Furthermore, let us focus on the case $R = 0$ which was shown to be representative of the more asynchronous cases, and which is re-defined under heterogeneous F as $\frac{r^M}{r_j^F} = \lfloor \frac{r^M}{r_j^F} \rfloor, \forall j \in I$.

Remark 3. *The nature of the results under homogenous F policy remains unchanged under heterogeneous F policy. For example, the necessary and sufficient condition of the Battle scenario under patient players and $R = 0$ generalizes from equation* (24) *(in Appendix C), namely* $r^M(0) > \overline{r^M}(0) = \frac{a-b}{a-d} r^F \overset{(11)}{=} \frac{3}{2} r^F$, *to*

$$r^M(0) > \overline{r^M}(0) = \frac{a-b}{a-d} \sum_{j=1}^{J} f_j r_j^F \overset{(11)}{=} \frac{3}{2} \sum_{j=1}^{J} f_j r_j^F . \tag{18}$$

The sufficient conditions are modified analogously.

Proof. See Appendix J. □

For example, with two countries A and B, the former being double the size of the latter, the condition in (18) for the specific *Battle* game becomes $r^M(0) > \overline{r^M}(0) = r_A^F + \frac{1}{2} r_B^F$.

It is important to note however that these results assume that every F policymaker fully incorporates both the benefits and the costs of his over-expansionary actions on the Union. This may not be the case in the real world since the benefits of the fiscal stimulus accrue almost exclusively to the fiscally indisciplined country itself, whereas the costs in terms of tighter monetary policy are spread across all countries (see eg Masson and Patillo (2002)). Therefore, the smaller the extent to which each fiscally indisciplined country internalizes the cost borne by other members, the higher the sufficient degree of M commitment $\overline{r^M}$; the explicit modelling of which we intend to do in future research. Nevertheless, the policy implication would still apply, namely that to (attempt to) discourage F indiscipline a stronger M commitment must be implemented.[28]

[28]It may however be the case that even an infinitely strong M commitment, $r^M \to \infty$, of a fully patient common central bank, $\delta_M = 1$, does not uniquely ensure General Discipline in such a non-altruistic monetary Union featuring moral hazard (the EMU comes to mind as an example). This implies that other, more direct types of enforcement/punishment mechanisms may have to be used to discourage member countries from F indiscipline.

9. Empirical Evidence

Our analysis has several testable implications. They primarily relate to the strength of long-run M policy commitment r^M - the degree of explicitness with which M policy (inflation) targets are specified in the central banking legislation. In particular, under most (but not all) circumstances a higher r^M - a more explicit inflation target - is predicted to:

1) improve M policy outcomes by reducing the level and variability of inflation and enhancing policy credibility;

2) be negatively correlated to the degree of central bank goal independence (goal-CBI) as r^M is a substitute for M's patience (in reducing inflation and enhancing policy credibility);[29]

3) improve (discipline) F policy outcomes by reducing the size of budget deficits and the debt.

Let us discuss these in reverse order.

9.1. **Prediction 3).**

In our companion paper Hughes Hallett, Libich and Stehlik (2007b) we explicitly examine this hypothesis in a cross country setting. Carefully controlling for the effect of goal-CBI and various endogeneity issues, our preliminary results lend support to our prediction that a stronger M policy commitment (more explicit inflation target) Granger-causes lower deficits/debts. Let us here substitute this with a case study that describes developments in New Zealand after its pioneering adoption of explicit inflation targeting in 1990.

9.1.1. *M Commitment Disciplining F Policy: Case Study of New Zealand by Dr Don Brash*[30]

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> 'New Zealand provides an interesting case study illustrating the arguments in the article. We adopted a very strong commitment by the monetary authority, the Reserve Bank of New Zealand, when the Minister of Finance signed the first Policy Targets Agreement (PTA) with me as Governor under the new Reserve Bank of New Zealand Act 1989 early in 1990. The PTA required me to get inflation as measured by the CPI to between 0 and 2% per annum by the end of 1992, with the Act making it explicit that I could be dismissed for

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[29] This interpretation is based on the fact that goal independent central bankers have a longer term in office (see Mahadeva and Sterne (2000) or Waller and Walsh (1996)) and a longer optimizing horizon is likely to translate into more patient behaviour (Eggertsson and Le Borgne (2003)). Since δ_M is a parameter in the policymaker's objective function our results relate to *goal*-CBI, not *instrument*-CBI (on this distinction see Debelle and Fischer (1994)).

[30] Dr Brash was the Governor of the Reserve Bank of New Zealand during 1988-2002 in which period the Bank pioneered the explicit inflation targeting framework - later followed by a majority of industrialized countries.

> failing to achieve that goal unless I could show extenuating circumstances in the form, for example, of a sharp increase in international oil prices. At the time, inflation was running in excess of 5%.
>
> In the middle of 1990, the Government, faced with the prospect of losing an election later in the year, brought down an expansionary budget. I immediately made it clear that this expansionary fiscal policy required firmer monetary conditions if the agreed inflation target was to be achieved, and monetary conditions duly tightened.
>
> Some days later, an editorial in the "New Zealand Herald", New Zealand's largest daily newspaper, noted that New Zealand political parties could no longer buy elections because, when they tried to do so, the newly instrument-independent central bank would be forced to send voters the bill in the form of higher mortgage rates.
>
> I was later told by senior members of the Opposition National Party that the Bank's action in tightening conditions in response to the easier fiscal stance had had a profound effect on thinking about fiscal policy in both major parties in Parliament.
>
> Some years later, in 1996, the Minister of Finance of the then National Party Government announced that he proposed to reduce personal income tax rates subject to this being consistent with the Government's debt to GDP target being achieved, to the fiscal position remaining in surplus, and to the fiscal easing not requiring a monetary policy tightening. The Minister formally wrote to me asking whether tax reductions of the kind proposed would under the economic circumstances then projected, require me to tighten monetary conditions. Given how the Bank saw the economy evolving at that time, I was able to tell the Minister that tax reductions of the nature he proposed would not require the Bank to tighten monetary conditions in order to stay within the inflation target.'

9.2. **Prediction 2).** While there exist no index that would measure the inflation target's explicitness, the closest proxies are the two key features of that regime: the degrees of (goal) transparency and accountability that make it impossible for the inflation target to be frequently changed. Combining this with the interpretation of goal-CBI as a proxy for M's patience predicts a negative correlation between goal-CBI and accountability, in line with Briault, Haldane and King (1997), de Haan, Amtenbrink and Eijffinger (1999) and Sousa (2002). See Figure 7 for an illustration using recent data.

Despite the arbitrary nature of these constructed indices, this finding is robust. It has been obtained using differently constructed indices for different countries

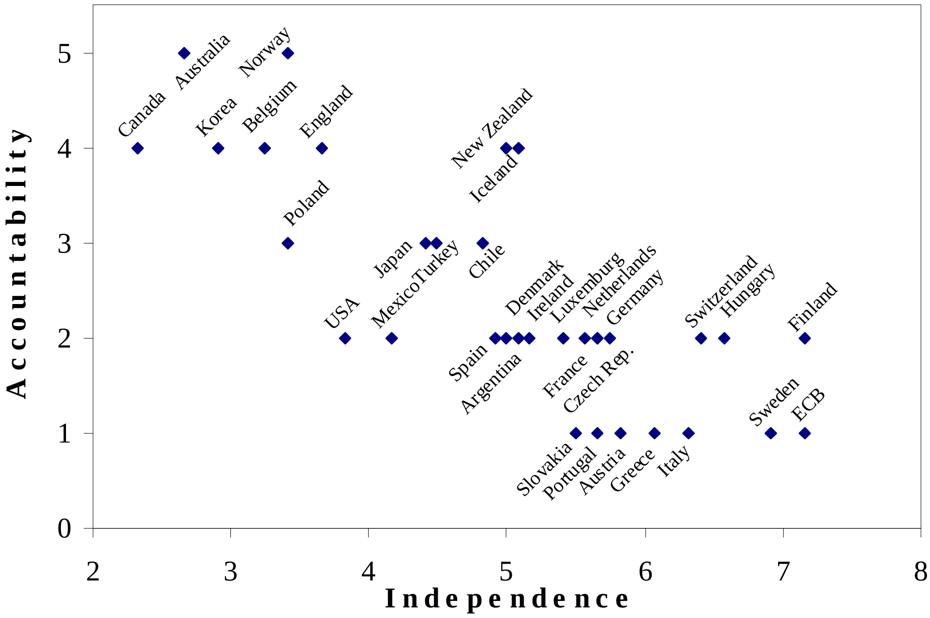

FIGURE 7. Central bank accountability vs independence using the Sousa (2002) indices. We depict the 'final responsibility' component of his accountability index here, see his paper for details on the criteria and scores. The correlation coefficient equals -0.78 (and the t-value equals 6.94).

and periods.[31] If we plot the Sousa (2002) final responsibility index against the length of term in office (which is one of the criteria in his CBI index) the picture remains roughly the same. Furthermore, in a comprehensive data set of Fry et al. (2000) the length of term in office is negatively correlated to accountability procedures in both industrial and transition countries. Finally, Hughes Hallett and Libich (2006) present evidence that transparency, too, is negatively correlated to goal-CBI. For example, it is shown that the correlation between transparency in

[31]Note that while all the countries in the top left hand corner are explicit inflation targeters, not all inflation targeters are in that corner – which is likely to be due to country specific factors. It should also be mentioned that this finding does not seem to be a result of omitted variables: all the countries in the sample have comparable inflation levels and existing economic theory does not identify any other reasons/variables for this negative relationship. In fact, the conventional view that accountability should go hand in hand with independence to be consistent with democracy (for a widely cited example see King (1998)) implies that the correlation should be positive.

Eijffinger and Geraats (2006) and goal-CBI in Briault, Haldane and King (1997) is -0.86 (and the t-value equals -4.46).[32]

9.3. **Prediction 1).** For the purposes of empirical testing it is important to note the exact nature of our results. The analysis implies that a more explicit long-run inflation target reduces the level of inflation and its volatility, but *only if* the initial level of explicitness is insufficient to uniquely achieve low and credible inflation (see Corollary 3). Otherwise r^M may have no long-run effect. Our results are therefore not equivalent to the claim that inflation targeting countries will have lower level and variability of inflation than non-targeting countries. This is because the latter group's implicit inflation target may have been sufficiently explicit: such that $r^M > \underline{r^M}(R)$.[33]

Our analysis implies a criterion to distinguish whether this is or isn't the case - it suggests to examine the *average* level of inflation (say over the past 5 years), $\bar{\pi}$. If $\bar{\pi} > \pi^L$ (arguably the case of many transition and developing countries) then $r^M < \overline{r^M}$ is implied and empirical tests will find the explicitness of inflation targeting to be negatively correlated with both the level of inflation and its volatility. In contrast, if $\bar{\pi} = \pi^L$ (the case for most industrial countries) then $r^M > \overline{r^M}$ is implied and our model predicts no correlation. Both predictions are supported in practice. Papers that only include industrial countries find weak or insignificant effects of inflation targeting on inflation and its volatility (Ball and Sheridan (2003) and Willard (2006)), whereas larger country samples find strong and significant effects (eg Corbo, Landerretche and Schmidt-Hebbel (2001)).

Furthermore, in line with the prediction of our model, inflation has been found negatively correlated with accountability (Briault, Haldane and King (1997)) and with transparency (Chortareas, Stasavage and Sterne (2002), Fry et al. (2000)). See also Debelle (1997) who finds inflation targeting to increase the policy's credibility. All these papers include either pre-1980 inflation data or developing countries. In contrast, papers that only focus on industrial countries and use recent data often find no correlation, see eg Eijffinger and Geraats (2006).

10. Robustness

This section briefly discusses some alternative specifications of commitment and implies that our results are robust.

[32] This paper also demonstrates that the Debelle and Fischer (1994) distinction between goal and instrument CBI is important. Since instrument CBI has come hand in hand with inflation targeting (as one of the prerequisites of the regime, see eg Masson, Savastano and Sharma (1997) or Blejer and et al. (2002)) its correlation with transparency and accountability is positive in most cases, see eg Chortareas, Stasavage and Sterne (2002).

[33] Also recall that even if $r^M < \overline{r^M}$ then MD *may* still obtain (in the region $\underline{r^M}(R) \leq r^M \leq \overline{r^M}$).

Endogenous Commitment. It should be noted that r^i can be endogenized as players' optimal choices (made at the very beginning of the game, in period 0). Libich (2007) is a step in this direction - in a different game it incorporates a cost of explicit commitment c^i (such as implementation or accountability cost) that is an increasing function of r^i. Naturally, whether any player finds it optimal to commit, and to what degree, will depend on the relative cost of doing so vis-à-vis the potential gain in terms of policy outcomes.[34]

Long-vs-short-run Commitment. One of the potential costs of commitment, for both M and F policy, is the potential reduction of the policy flexibility to react to shocks and hence stabilize output. For example in M policy, these concerns were spelled out by inflation targeting opponents such as Kohn (2003), Friedman (2004) and Greenspan (2003)). To examine these in detail our companion paper Libich (2006) utilizes the asynchronous framework using a 'stochastic' New Keynesian type environment. It shows that allowing for disturbances does not alter the conclusions of the presented paper if the inflation target is specified as a long-run objective (achievable on average over the business cycle - the case in most industrial countries).[35] The same can be (and has been) argued about F policy: a long-run balanced budged run only restricts average levels, not the possibility of ever having a deficit implied by the business cycle.

In the terminology of Kydland and Prescott (1977), long-run policy commitment may be consistent with short-run discretion. Put differently, the 'credibility vs flexibility tradeoff' (see eg Lohmann (1992)) relates to a short-run, not a long-run commitment mechanism.

Probabilistic Commitment. If deterministic commitment of Taylor (1980) is reinterpreted as a probabilistic one in the spirit of the Calvo (1983), see (5) in Section 3, then the average/expected length of time between each move is $\frac{1}{1-\theta^i}$. This is equivalent to our deterministic r^i and hence we would expect similar findings. Such conjecture is supported in a companion paper Libich and Stehlik (2007b) in which we examine this probabilistic version explicitly. We show that the intuition remains the same, ie under a sufficiently committed and patient M policymaker, $\theta^M > \overline{\theta^M}$ and $\delta_M > \overline{\delta_M}$, General Discipline uniquely obtains.

Continuous Commitment. Libich and Stehlik (2007b) present analogous results for continuous time, $t \in \mathbb{R}$, which can incorporate not only the players

[34]And one would expect these costs to differ; the cost of explicitly committing to a long-run inflation target is arguably small relative to the (political) cost of explicitly committing to a long-run balanced budget rule.

[35]The paper in fact finds the opposite, the policymaker's flexibility under an explicit long-run IT is likely to increase which reduces the volatility of both inflation and output in equilibrium. This is due to the 'anchoring' effect of ITs that has been found empirically (eg Gurkaynak et al (2005)) and that our asynchronous framework enables us to model explicitly. For arguments and results in the same spirit see Orphanides and Williams (2005), Bernanke (2003), Goodfriend (2003) and Mishkin (2004).

heterogeneity but also the probabilistic models. Roughly speaking, if we denote by $f : [0, r^M] \to [0,1]$ a non-decreasing function which describes a distribution of various F's reactions, then the necessary and sufficient condition analogous to (24), $r^M(0) > \frac{a-b}{a-d} r^F \stackrel{(11)}{=} \frac{3}{2} r^F$ in Appendix C, is

$$\int_0^{r^M} f(t)dt > \frac{a-b}{a-d} r^F \stackrel{(11)}{=} \frac{3}{2} r^F. \tag{19}$$

Time-varying Commitment. Both continuous and discrete models (and all of our above results) can be neatly unified and extended using time scales calculus - a recent mathematical tool (see Bohner and Peterson (2001) for a comprehensive treatment). The main contribution of this environment for our purposes is the ability to consider *non-constant* (time-varying) commitment. This generalization is arguably realistic and hence important in many settings in economics, econometrics, as well as other disciplines.[36]

A *time scale* $\mathbb{T}$ is defined as a nonempty closed subset of the real numbers $\mathbb{R}$. In the analysis, the so-called 'jump operators' play a key role that discribe the varying time steps. Libich and Stehlik (2007b) show that the condition analogous to (19) and (24) is

$$\int_0^{r^M} f(t)\Delta t > \frac{a-b}{a-d} r^F \stackrel{(11)}{=} \frac{3}{2} r^F. \tag{20}$$

where the LHS is called 'delta integral' such that

$$\int_0^{r^M} f(t)\Delta t = \begin{cases} \int_0^{r^M} f(t)dt & \text{if} \quad \mathbb{T} \in \mathbb{R}, \\ \sum_{t=0}^{r^M-1} f(t) & \text{if} \quad \mathbb{T} \in \mathbb{Z}. \end{cases} \tag{21}$$

This shows that since time scale calculus nests both continuous and discrete time as special cases it allows for even more flexible analysis of *dynamic interactions* with heterogeneous time steps.

11. Summary and Conclusions

The current stance of fiscal policy in a number of countries (including the US and EU) has raised concerns about the degree of discipline in fiscal policies, and about the risks for the credibility and outcomes of monetary policy. This paper

[36] For an interesting application of time scales in economics see Biles, Atici and Lebedinsky (2005). The authors model payments to an agent (eg capital income or dividents) arriving at unevenly spaced intervals.

highlights the importance of understanding the monetary-fiscal interactions and the effect of various commitment arrangements (policy rules) in assessing whether this poses a problem.

Specifically, to contribute to this debate we propose a novel asynchronous game theoretic framework that generalizes the standard commitment concept in a number of respects. Most importantly, it allows for: (i) concurrent commitment of more than one player/policy, (ii) partial commitment, and (iii) endogenously determined (optimally selected) commitment.

Our analysis shows that the effect of commitment on economic outcomes of the policy interaction crucially depends on the *type* of commitment, and on the *relative degrees* of monetary and fiscal commitment. In particular, it is first shown that the 'fiscal concerns' are justified since inflation bias and lack of credibility may still hold in equilibrium even under a fully independent, responsible, patient, and committed central banker.

Nevertheless, it is also demonstrated that this undesirable scenario can be prevented if monetary policy commitment is *sufficiently* strong - above a certain threshold. This threshold degree is function of the policymakers' discount factors, conservatism and the structure of the economy. Furthermore and interestingly, it is shown that such monetary commitment can not only resist the fiscal pressure, but also indirectly (through incentives) 'discipline' an ambitious fiscal policymaker and achieve socially desirable outcomes for *both* policies. It should be noted that our commitment is to the regime itself and hence long-run outcomes, rather than to specific short-run policies within it, which still allows for flexibility to stabilize shocks.

The implication for monetary policymakers (in countries with ambitious fiscal policymakers which arguably currently includes the US and EU) is that to discourage and/or counteract over-expansionary fiscal policies they should make their inflation target more explicit. The implication for fiscal policymakers is that imposing monetary commitment (eg legislating a numerical inflation target) may provide a way to indirectly tie their hands if direct fiscal reform seems politically infeasible. Our analysis has a number of predictions that we show to be empirically supported.

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Appendix A. Proof of Proposition 1

Proof. In terms of claim (i), inspection of (8) suggests that for all values except $\rho = \mu$ we have π_t^* to be a function of G_t. Claim (ii) shows that if unless $x_T^F = x_T^M = 0$ we obtain $\pi_t^* \neq 0$ and hence $C_t < 0$ where $\pi_t^* > 0$ obtains (under $x_T^F > x_T^M \wedge \rho > \mu$ or $x_T^F < x_T^M \wedge \rho < \mu$) or $\pi_t^* < 0$ obtains (under $x_T^F > x_T^M \wedge \rho < \mu$ or $x_T^F < x_T^M \wedge \rho > \mu$). Claim (iii) is implied by (8) which shows that under $\rho < \mu$, π_t^* is decreasing in both β^M and x_T^M. □

Appendix B. Proof of Proposition 2

Proof. To prove the existence claims it suffices to derive parameter values under which each scenario obtains. Such examples are reported in Figures 8 and 9 that show all five feasible scenarios.

In terms of the non-existence of the *M-gap* and *Chicken* scenarios recall from Definition 3 that in both (MI, FD) is a Nash equilibrium. For this to be the case it must be true that $FD \in b(MI)$ and $MI \in b(FD)$. Using the reaction functions in (7) with the definition of $\{MD, MI, FD, FI\}$ in (9), $MI \in b(FD)$ requires $\pi^* = -\frac{x_T^F}{\rho} = -\frac{x_T^M \beta^M(\rho-\mu)}{1+\beta^M \rho(\rho-\mu)}$. This, after rearranging, yields

$$x_T^M = x_T^F \left(1 + \frac{1}{\beta^M \rho(\rho - \mu)}\right) \quad \text{for all} \quad \beta^M \rho(\rho - \mu) \neq -1. \tag{22}$$

It is apparent that, under $x_T^F > 0$ and $x_T^M = 0$, there are no parameter values that satisfy (22). □

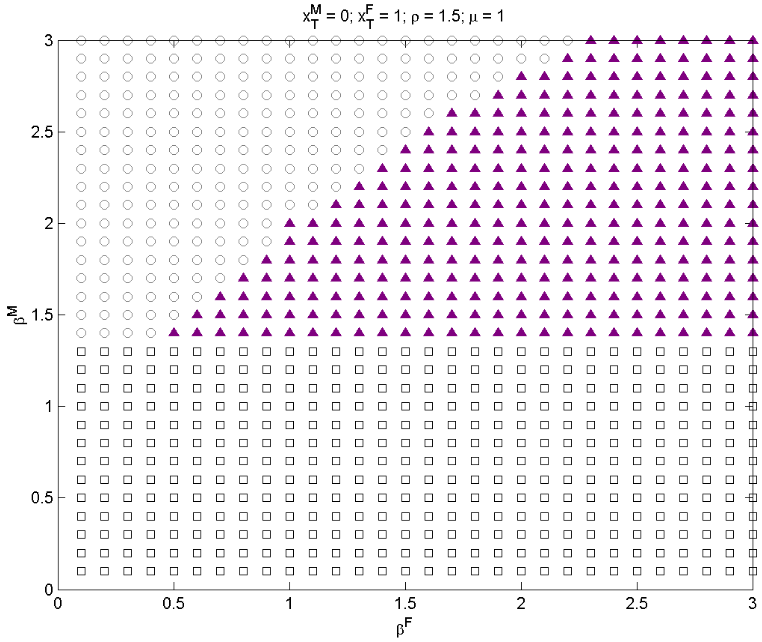

FIGURE 8. Outcomes under $x_T^M = 0, x_T^F = 1, \rho = 1.5, \mu = 1$ and various β^M and β^F. The symbols denote the following scenarios: square: *M&F-gap*; pyramid: *Battle*; circle: *Coordination*.

APPENDIX C. PROOF OF PROPOSITION 3

Proof. We solve the game backwards and prove the statement by a mathematical induction argument with respect to M's moves, restricting our attention to the relevant region $r^M > r^F$.

First, we prove that on the equilibrium path D will be played in M's last move $n^M = N^M$ (the inductive basis) and then, supposing that it holds for some $n^M \leq N^M$, we show that the same is true for $n^M - 1$ as well. Put differently, all n^M moves will then be history-independent. This will prove that on the equilibrium path of any SPNE we have $M_n^D, \forall n^M$. Since F's unique best response to MD is FD, it will follow that in equilibrium $F_n^D, \forall n^F$.[37]

A) $\mathbf{n^M = N^M}$ **under** $\mathbf{R = 0}$: Focusing first on this special case is illustrative. Here we have, due to $r^M > r^F$, $T(r^M, r^F) = r^M$ and therefore $N^M = 1$ (and $N^F = r^M$). Solving backwards, we know that $F_{n>1}^* \in b(M_1)$ due to perfect information in $n^F > 1$. Further, from F's rationality and complete information it follows that $F_1^* \in b(M_1)$. For there to exist only the Disciplined type of SPNE, it is therefore

[37]It will become evident that for most parameter values satisfying (12) there will be a *unique* Disciplined SPNE. Nevertheless, since our attention will be on the equilibrium path we will not examine the exact number of SPNE (off-equilibrium behaviour).

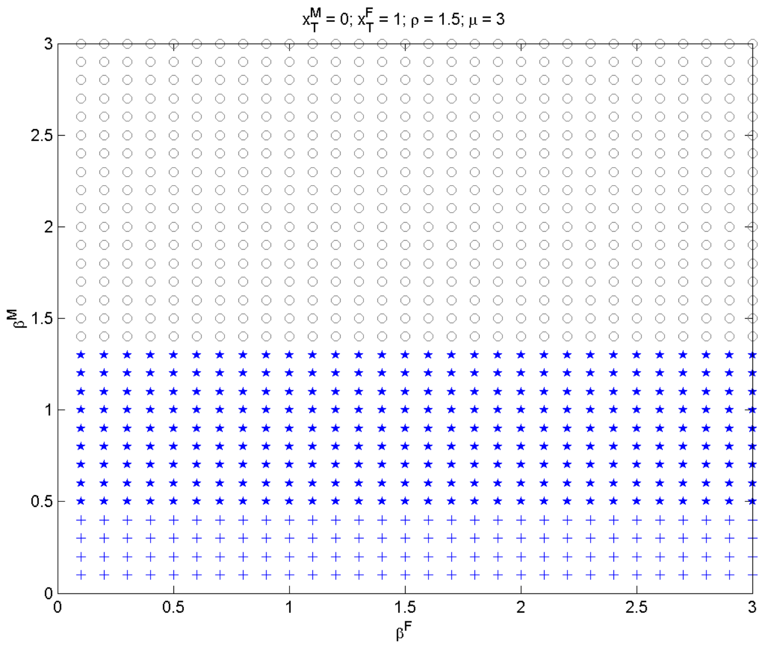

FIGURE 9. Outcomes under $x_T^M = 0, x_T^F = 1, \rho = 1.5, \mu = 3$ and various β^M and β^F. The symbols denote the following scenarios: circle: *Coordination*; star: *No-gap*; cross: *F-gap*.

required that $b(F_1^I) = \{M_1^D\}$ which yields the following condition

$$br^F + a(r^M - r^F) > dr^M. \tag{23}$$

The left-hand side (LHS) and right-hand side (RHS) of (23) report M's payoffs from playing D and I respectively. Since (23) assumes F_1^I then if M is disciplined, M_1^D, the monetary policymaker will initially suffer higher variability of output (payoff b). This however only lasts for r^F periods after which F finds it optimal to switch to F^D, which 'rewards' M for its discipline by the stability benefit, payoff a, for the rest of the unrepeated game, $(r^M - r^F)$ periods. Intuitively, (23) expresses that, for π^D to be played, this reward has to more than offset the initial loss. Rearranging (23) then yields

$$r^M(0) > \overline{r^M}(0) = \frac{a-b}{a-d} r^F \overset{(11)}{=} \frac{3}{2} r^F, \tag{24}$$

where, as defined in Section 3.4, $\overline{r^M}(0)$ is a *necessary and sufficient* degree of M commitment for the case $R = 0$.

B) $\mathbf{n^M = N^M}$ **under** $\mathbf{R > 0}$: From Definition 2 it follows that the number of M's moves is $N^M = \frac{T(r^M, r^F)}{r^M} > 1$. A condition analogous to (23) is the following

$$br^F R + a(r^M - r^F R) > dr^M. \tag{25}$$

Rearrange this to obtain

$$r^M > \frac{a-b}{a-d} R r^F, \tag{26}$$

This means that, if (26) holds, a patient M will find it optimal to play M_N^D for all histories.

C) $\mathbf{n^M + 1 \to N^M}$ (if applicable, ie if $1 \leq n^M < N^M$): The proof proceeds by induction. We first assume that M's unique best play in the $(n^M + 1)$-th step is MD regardless of F's preceding play (ie that M_{n+1} is history-independent), and we attempt to prove that this implies the same assertion for the n^M-th step. Intuitively, this means that if M inflates he finds it optimal to immediately disinflate. Two scenarios are possible in terms of the underlying fiscal behaviour since that will determine the costs of the disinflation. If F runs a growing debt, F^I, the payoffs b and w will occur for at least one period, whereas if F runs a stable debt, F^D, the disinflation will only be accompanied by the payoffs a and v (note that in the former case the disinflation is more *costly* to both policymakers since $a > b$ and $v > w$ from (10)). This implies that one of the following two conditions, analogous to (23), will apply at move n^M

$$bk_n + a(r^M - k_n) + a[r^F - (r^F - k_{n+1})] > dr^M + b[r^F - (r^F - k_{n+1})], \tag{27}$$

and

$$bk_n + a(r^M - k_n) > d[r^M - (r^F - k_{n+1})] + a(r^F - k_{n+1}). \tag{28}$$

Which of these two conditions is relevant to a certain n^M depends on F's payoffs $\{v, w, y, z\}$, and importantly on k_{n+1}. Specifically, if

$$(r^F - k_{n+1})z + k_{n+1}w \geq (r^F - k_{n+1})y + k_{n+1}v, \tag{29}$$

then (27) obtains, otherwise (28) is the relevant condition.[38]

Now, we will show that if the conditions (27) and (28) are satisfied at $n^M = 1$, then they hold in all other n^M as well. This convenient feature notably simplifies the solution of the game.

Lemma 1. *Consider the Battle scenario in which* (10) *holds and* $\delta_F = \delta_M = 1$. *Then for all* R *the necessary and sufficient conditions to uniquely ensure General Discipline are obtained at* $n^M = 1$ *(the initial simultaneous move).*

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[38]For the specific game this condition becomes $k_{n+1} \leq \frac{z-y}{z-y+v-w} \overset{(11)}{=} \frac{4}{5} r^F$. This implies that in the game in Figure 3 with $r^F = 3$ and $r^M = 5$, all disinflations (in $n^M \geq 2$) would be costly and (27) would apply. We will see below that the parameter space under which (28) obtains gets smaller with the F's impatience.

Proof. Equations (27) and (28) can be, respectively, rearranged into

(30) $$r^M > \frac{a-b}{a-d}(k_n - k_{n+1}),$$

(31) $$r^M > \frac{(a-b)k_n}{a-d} + (r^F - k_{n+1}).$$

The strength of both conditions is increasing in k_n and decreasing in k_{n+1}. Thus the strongest condition is guaranteed by the maximum of the difference $(k_n - k_{n+1})$. From (6) it follows that $k_n - k_{n+1} \leq Rr^F$. The fact that $k_1 - k_2 = Rr^F$ then proves the claim. □

Continuing the proof of Proposition 3, this property means that regardless of the exact dynamics/asynchronicity, it suffices to focus on the initial simultaneous move (similarly to a one-shot game) *assuming* that all further relevant conditions hold. If the strongest condition for $n^M = 1$ is satisfied we then know that a unique (type of) equilibrium outcome obtains throughout. Using the implied $k_1 = r^F$ and $k_{n+1} = k_n - Rr^F$ jointly yields $k_2 = (1-R)r^F$. Substituting these into (27)-(28) or (30)-(31) we obtain, together with (24)

(32)
$$r^M > \overline{r^M}(R) = \begin{cases} \frac{a-b}{a-d} r^F \overset{(11)}{=} \frac{3}{2} r^F & \text{if} \quad R = 0, \\ \frac{a-b}{a-d} Rr^F \overset{(11)}{=} \frac{3}{2} Rr^F & \text{if} \quad R > \bar{R} = \frac{v-w}{z-y+v-w} \overset{(11)}{=} \frac{1}{5}, \\ \left(\frac{a-b}{a-d} + R\right) r^F \overset{(11)}{=} \left(\frac{3}{2} + R\right) r^F & \text{if} \quad R \leq \bar{R} = \frac{v-w}{z-y+v-w} \overset{(11)}{=} \frac{1}{5}, \end{cases}$$

where the threshold $\bar{R} \in (0,1)$ is implied by (29). These inequalities together with (24) (for $R = 0$) are the three necessary and sufficient conditions for uniqueness of the Disciplined type of SPNE (note that all three are at least as strong as the condition for N^M in (26)). Combining these three conditions implies the sufficient conditions (12) and (13), and completes the proof of Proposition 3. □

Appendix D. Proof of Proposition 4

Proof. Under $R = 0$ the value of δ_F does not affect the relevant sufficient condition in (24). However, if $R = (0,1)$ and F is sufficiently impatient, $\delta_F \leq \overline{\delta_F}$, where the threshold value $\overline{\delta_F}$ is a function of $\{r^M, r^F, v, w, y, z\}$, the sufficient condition will alter. Instead of deriving analytically $\overline{\delta_F}$ from (29) we focus on the extreme case $\overline{\delta_F} = \delta_F = 0$ which is a sufficiently low threshold for all r^M, r^F and for all $\{v, w, y, z\}$ satisfying (10).

The impatient F will disregard the future and always play $G_t^* \in b(\pi_t)$. Since for all but the initial move the policymakers never move simultaneously this implies $F_{n>1}^* \in b(\pi_{t-1})$. Intuitively, a sufficiently impatient F will never reduce G before the start of disinflation and hence disinflation will always be costly for both players.

Formally, (28) no longer applies and (27) becomes the relevant condition $\forall n^M, R \in (0,1)$, and for all $\{a,b,c,d,v,w,y,z\}$ satisfying (10).

Hence we need to show that any $r^M > r^F$ satisfy the following two conditions: (i) under $R = 0$ it holds that $\frac{r^M}{r^F} > \frac{a-b}{a-d}$ (from (24)) and (ii) $\forall R \in (0,1)$ it is true that $\frac{r^M}{r^F} > \frac{a-b}{a-d}R$ (from (32)). To prove (ii) note that any $r^M > r^F$ has, from the definition of R, the property that $\frac{r^M}{r^F} \geq 1 + R$. Therefore claim (ii) can be rewritten as $1 + R > \frac{a-b}{a-d}R$. Divide both sides by R to obtain $\frac{1}{R} + 1 > \frac{a-b}{a-d}$. To see that this is satisfied utilize two characteristics. First, $\frac{1}{R} + 1 > 2$ since $R < 1$. Second, rearrange $a > 2d - b$ into $2 > \frac{a-b}{a-d}$. Combining these gives $\frac{1}{R} + 1 > 2 > \frac{a-b}{a-d}$ which completes the proof of (ii). To show (i), note that under $R = 0$ all $r^M > r^F$ satisfy $\frac{r^M}{r^F} \geq 2$. Using this jointly with $2 > \frac{a-b}{a-d}$ completes the proof. □

Appendix E. Proof of Proposition 5

Let us first extend the result of Lemma 1 under impatience.

Lemma 2. *Consider the Battle scenario in which* (10) *holds. Then* $\forall \delta_M, \delta_F$, *and* R, *the necessary and sufficient conditions to uniquely ensure General Discipline are obtained at* $n^M = 1$ *(the initial simultaneous move).*

Proof. Lemma 1 shows this claim to hold under $\delta_M = \delta_F = 1$. The proof of Proposition 4 showed that δ_F affects whether (27) or (28) applies in some n^M, but not the implication of (30)-(31) that they are both the strongest at $n^M = 1$. Let us therefore consider the effect of M's impatience. Under $\delta_M < 1$ the inequality in (27), that applies to the case $R > \bar{R}$, becomes

$$b\sum_{t=1}^{k_n}\delta_M^{t-1} + a\sum_{t=k_n+1}^{r^M}\delta_M^{t-1} + a\sum_{t=r^M+1}^{r^M+k_{n+1}}\delta_M^{t-1} > d\sum_{t=1}^{r^M}\delta_M^{t-1} + b\sum_{t=r^M+1}^{r^M+k_{n+1}}\delta_M^{t-1}. \tag{33}$$

This can be rearranged into

$$(d-b)\sum_{t=1}^{k_n}\delta_M^{t-1} - (a-d)\sum_{t=k_n+1}^{r^M}\delta_M^{t-1} - (a-b)\sum_{t=r^M+1}^{r^M+k_{n+1}}\delta_M^{t-1} < 0.$$

Use $a - b = (a-d) + (d-b)$ and split the first series to obtain

$$(a-b)\sum_{t=1}^{k_n}\delta_M^{t-1} - (a-d)\sum_{t=1}^{r^M}\delta_M^{t-1} - (a-b)\sum_{t=r^M+1}^{r^M+k_{n+1}}\delta_M^{t-1} < 0,$$

add $\sum_{t=k_n+1}^{r^M}\delta_M^{t-1}$ to both sides and collect terms

$$(a-b)\sum_{t=1}^{r^M}\delta_M^{t-1} - (a-d)\sum_{t=1}^{r^M}\delta_M^{t-1} < (a-b)\sum_{t=k_n+1}^{r^M+k_{n+1}}\delta_M^{t-1}.$$

Adding up the terms on the RHS we obtain

$$(a-b)\sum_{t=1}^{r^M}\delta_M^{t-1}-(a-d)\sum_{t=1}^{r^M}\delta_M^{t-1}<(a-b)\delta_M^{k_n}\frac{1-\delta_M^{r^M+k_{n+1}-k_n}}{1-\delta_M}.$$

Since $\delta_M<1$ we see that, analogously to Lemma 1, the strength of the condition is increasing in k_n and decreasing in k_{n+1}. Hence the same argument applies. We can also see that, for $R\le\bar{R}$ (using (28)), the effect of M's impatience is analogous. Finally, for $R=0$ we have $N^M=1$ which finishes the proof. □

Using this convenient property let us continue the proof of Proposition 5.

Proof. Claim (i): It is apparent in (32) that the *strongest* possible necessary and sufficient condition (highest $\overline{r^M}(R)$) obtains under *costless* disinflation if F's payoffs $\{v,w,y,z\}$ are such that $\bar{R}\to 1$ (since the inflation cost d lasts the shortest period of time). Furthermore, we have shown in Proposition 4 that the opponent's impatience weakens the sufficient conditions. Therefore, we can focus on the analog of (27) under $0\le\delta_M<1=\delta_F$, which is

$$b\sum_{t=1}^{k_n}\delta_M^{t-1}+a\sum_{t=k_n+1}^{r^M}\delta_M^{t-1}>d\sum_{t=1}^{r^M-r^F+k_{n+1}}\delta_M^{t-1}+a\sum_{t=r^M-r^F+k_{n+1}+1}^{r^M}\delta_M^{t-1}. \tag{34}$$

Now use the fact that this condition is the strongest for $k_n=r^F$ and $k_{n+1}\to 0$ (the latter follows from $\bar{R}\to 1$), and rearrange to obtain

$$(a-d)\sum_{t=r^F+1}^{r^M-r^F}\delta_M^{t-1}>(d-b)\sum_{t=1}^{r^F}\delta_M^{t-1}. \tag{35}$$

It therefore suffices to show that the condition of Proposition 5, namely (15), implies (35). To do so note that (15) can be rearranged into

$$\delta_M^{r^F}>\frac{d-b}{a-b},$$

which can be manipulated to give

$$0<1-\frac{d-b}{a-d}\frac{1-\delta_M^{r^F}}{\delta_M^{r^F}}.$$

Since $\delta_M^{2r^F}>0$, it is true that

$$0<\delta_M^{2r^F}\left(1-\frac{d-b}{a-d}\frac{1-\delta_M^{r^F}}{\delta_M^{r^F}}\right).$$

Consequently, for each $\delta_M = (0,1)$ there exists $\overline{r^M} \in \mathbb{N}$ such that for all $r^M > \overline{r^M}$

$$\delta_M^{r^M} < \delta_M^{2r^F} \left(1 - \frac{d-b}{a-d}\frac{1-\delta_M^{r^F}}{\delta_M^{r^F}}\right).$$

Multiplying both sides by $-(a-d)\delta_M^{r^F} > 0$ and dividing by $\delta_M^{2r^F}$ we obtain

$$(a-d)\delta_M^{r^F}\left(1-\delta_M^{r^M-2r^F}\right) > (d-b)(1-\delta_M^{r^F}).$$

Moreover, we divide both sides by $1-\delta_M > 0$

$$(a-d)\delta_M^{r^F}\frac{1-\delta_M^{r^M-2r^F}}{1-\delta_M} > (d-b)\frac{1-\delta_M^{r^F}}{1-\delta_M}.$$

Note that the two fractions are in fact partial sums of geometric series with quotient δ_M as follows

$$(a-d)\sum_{t=r^F+1}^{r^M-r^F}\delta_M^{t-1} > (d-b)\sum_{t=1}^{r^F}\delta_M^{t-1},$$

which is the desired condition in (35).

Claim (ii): It is apparent in (32) that the *weakest* possible necessary and sufficient condition (lowest $\overline{r^M}(R)$) obtains under *costly* disinflation if F's payoffs $\{v,w,y,z\}$ are such that $\bar{R} \to 0$ (since the disinflation cost b lasts the longest period of time). Furthermore, we have shown in Proposition 4 that the opponent's impatience weakens the sufficient conditions. Therefore, we can focus on the analog of (27) under $0 = \delta_F \leq \delta_M < 1$, (33), imposing the implication of Lemma 2 that $k_{n+1} = k_2 \to k_n = k_1 = r^p$ (the latter leading to $\bar{R} \to 0$). Substituting this into (33) yields

$$(a-d)\sum_{t=r^F+1}^{r^M}\delta_M^{t-1} + (a-b)\sum_{t=r^M+1}^{r^M+r^F}\delta_M^{t-1} > (d-b)\sum_{t=1}^{r^F}\delta_M^{t-1}. \tag{36}$$

Using the formula for a finite sum of geometric series and rearranging yields

$$(1-\delta_M^{r^F})[(a-b)\delta_M^{r^F} - (d-b)] \overset{(11)}{=} (1-\delta_M^{r^F})(2\delta_M^{r^F}-1) > 0.$$

This is *not* satisfied are all values $\delta_M \leq \overline{\delta_M}$ where the threshold is from (15). The fact that there may be no Disciplined SPNE implies that the inequality must be strict, which completes the proof. □

Appendix F. Proof of Proposition 6

Proof. Under M's impatience, the condition analogous to (23) becomes

$$b\sum_{t=1}^{r^F}\delta_M^{t-1} + a\sum_{t=r^F+1}^{r^M}\delta_M^{t-1} > d\sum_{t=1}^{r^M}\delta_M^{t-1}. \tag{37}$$

which can be, using the formula for a sum of a finite series, rewritten as

$$b\frac{1-\delta_M^{r^F}}{1-\delta_M} + a\delta^{r^F}\frac{1-\delta_M^{r^M-r^F}}{1-\delta_M} > d\frac{1-\delta_M^{r^M}}{1-\delta_M}.$$

By analyzing this equation we observe that (37) holds if and only if (16) is satisfied. Now we can utilize two properties which follow from (16). First, the argument of the logarithm in (16) is positive if and only if $\delta_M > \overline{\delta_M}$ (from (15)) holds. Second, both the base and the argument of the logarithm in (16) lie strictly between 0 and 1. To see this, recall that

$$0 < \frac{a-b}{a-d}\delta_M^{r^F} - \frac{d-b}{a-d} < \frac{a-b}{a-d} - \frac{d-b}{a-d} = 1.$$

Therefore $\overline{r^M}(0)$ is positive and increasing in r^F. In order to prove the substitutability claim, take (16) and rewrite it as

$$\overline{r^M} = \frac{\ln(\frac{a-b}{a-d}\delta_M^{r^F} - \frac{d-b}{a-d})}{\ln \delta_M}.$$

Our task now is to show that $\overline{r^M}$ is decreasing in δ_M on the considered domain

$$D := \left(\sqrt[r^F]{\frac{d-b}{a-b}}, 1\right). \tag{38}$$

For the sake of clarity, we simplify the notation by defining

$$\gamma := \frac{a-b}{a-d},\ \omega := \frac{d-b}{a-d},\ r := r^F, \delta := \delta_M.$$

Therefore, we want to show that the function

$$f(\delta) = \frac{\ln(\gamma\delta^r - \omega)}{\ln \delta}$$

is decreasing in δ, or equivalently that $f'(\delta) < 0$ on D. Obviously,

$$\begin{aligned} f'(\delta) &= \frac{\frac{r\gamma\delta^{r-1}}{\gamma\delta^r-\omega}\ln\delta - \frac{1}{\delta}\ln(\gamma\delta^r-\omega)}{\ln^2\delta} \\ &= \frac{r\gamma\delta^r\ln\delta - (\gamma\delta^r-\omega)\ln(\gamma\delta^r-\omega)}{\delta(\gamma\delta^r-\omega)\ln^2\delta}. \end{aligned}$$

Since the denominator is always positive, it suffices to show that

$$r\gamma\delta^r\ln\delta - (\gamma\delta^r-\omega)\ln(\gamma\delta^r-\omega) < 0,$$

or equivalently

$$\phi(\delta) := r\gamma\delta^r\ln\delta < (\gamma\delta^r-\omega)\ln(\gamma\delta^r-\omega) =: \psi(\delta)$$

on the considered domain, D from (38). Taking into account the definitions of γ and ω, we observe that $\phi(1) = 0 = \psi(1)$. Therefore it suffices to show that $\phi'(\delta) > \psi'(\delta)$ for all $\delta \in D$. But this is satisfied since:

$$\begin{aligned}
\phi'(\delta) &> \psi'(\delta) \\
r^2\gamma x^{r-1}\ln\delta + r\gamma x^{r-1} &> r\gamma x^{r-1}\ln(\gamma\delta^r - \omega) + r\gamma x^{r-1} \\
r\ln\delta &> \ln(\gamma\delta^r - \omega) \\
\delta^r &> \gamma\delta^r - \omega \\
\delta_M^{r^F} &> \frac{a-b}{a-d}\delta_M^{r^F} - \frac{d-b}{a-d} \\
(a-d)\delta_M^{r^F} &> (a-b)\delta_M^{r^F} - (d-b) \\
0 &> (d-b)(\delta_M^{r^F} - 1),
\end{aligned}$$

where the last inequality is trivially satisfied since $d > b$ and $\delta_M < 1$. □

Appendix G. Proof of Corollary 3

Proof. It follows from Proposition 2 that if $r^M < \underline{r^M}$ then the level π^I obtains *uniquely* in equilibrium, ie π^D is never M's optimal play. This implies time-inconsistency of the inflation target, lack of its credibility, and an equilibrium inflation bias. Furthermore, it implies growing debt, both in nominal and real terms, $G^* > g^* > 0$. In contrast, it was shown in Proposition 5 that under $r^M > \overline{r^M}$ there uniquely exists General Discipline, ie inflation and the growth of (both nominal and real) debt are zero. □

Appendix H. Proof of Proposition 7

Proof. To prove these existence claims it suffices to provide specific examples. For the reader's convenience we will consider the specification of Figures 2-3, namely $r^M = 5, r^F = 3$, and the values of the specific *Battle* game (11) with only one change: the cost of disinflation for M, payoff b, will be made greater and re-set to $b = -3$. Let us first show that there exists no Disciplined SPNE and then that an Indisciplined SPNE exists.

Focus on the condition for M's last move, $n^M = N^M$, to be uniquely π^D in equation (26), $\frac{r^M}{r^F} > \frac{a-b}{a-d}R$. Notice that, under these specific circumstances, this condition is not satisfied, $\frac{5}{3} \not> 2$. Therefore, M_3 is no longer history-independent and M_3 will be the best response to F's preceding move, F_4. Moving forward, the player F takes this into account in comparing the continuation payoffs from F_4^D and F_4^I. If M_2^D then F's continuation payoff from playing F_4^D is $v[(1-R)r^F + r^M] = 0$ whereas from playing F_4^I is $w(1-R)r^F + zr^M = \frac{9}{2}$. Therefore, F_4 is now history-independent - regardless of M's preceding move, M_2, F will uniquely play F_4^I in order to ensure the D levels for the rest of the unrepeated game. This proves that

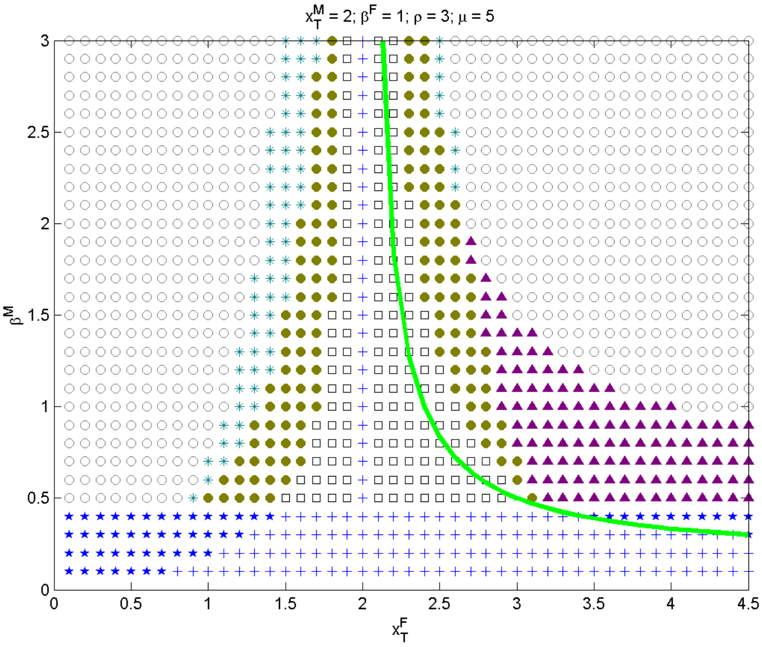

FIGURE 10. Outcomes under $x_T^M = 2, \beta^F = 1, \rho = 3, \mu = 5$ and various β^M and x_T^F. The symbols denote the following scenarios: square: *M&F-gap*; pyramid and full circle: *Battle* (in the latter M prefers General Indiscipline); empty circle and asterisk: *Coordination* (in the latter both players prefer General Indiscipline); star: *No-gap*; cross: *F-gap*; and the thick line: *M-gap*.

in this case there exists no Disciplined SPNE as there will never be F_4^D on the equilibrium path.

In order to prove that there exists an Indisciplined SPNE we need to show that in M_2 the level D is not a unique play regardless of the level played in F_2 (for M_1 this is automatically satisfied since the move is simultaneous and $r^M > r^F$). Therefore, move forward and assume F_2^I. Then, using the above information, M's continuation payoff from playing M_2^D is $b(1-R)r^F + ar^F + b(1-R)r^F + dr^M = -3$ whereas from playing M_2^I is $2dr^M = 0$. Comparing these two implies that M_2^* will be the best response to F_2 and hence an Indisciplined SPNE exists. □

APPENDIX I. PROOF OF PROPOSITION 8

Proof. To proof these existence claims, it suffices to derive parameter values under which each scenario obtains. Figure 10 reports parameter values under which *all* of the scenarios (except the infeasible *Chicken*) obtain.

In comparison to the case $x_T^M = 0$ reported in Proposition 2, there is an additional scenario, *M-gap*. The proof of that proposition showed that this scenario obtains if (22) is satisfied which can be the case under $x_T^M > 0$. In terms of the non-existence of the *Chicken* scenario recall from Definition 3 that it also requires the (MD, FI) to be a Nash equilibrium. For this to be the case it must be true that $FI \in b(MD)$ and $MD \in b(FI)$. Using the reaction functions in (7) this requires $\rho = \mu$ or $x_T^M = x_T^F$. It is clear that the *Chicken* scenario does not obtain since under neither of these conditions can (22) be satisfied. That is to say, there exist no parameter values under which both (MD, FI) and (MI, FD) are Nash equilibria. □

Appendix J. Proof of Remark 3

Proof. Note that $R = 0$ implies $T(r^M, r_j^F) = r^M$ and therefore $N^M = 1$. Solving backwards, as in Section 6.1 we know that $(F_{n>1}^j)^* \in b(M_1), \forall j$ and $(F_1^j)^* \in b(M_1), \forall j$. This implies that all $j \in I$ will select the same moves in all their n_j^F, ie $F_n^j = F_n, \forall n, j$.[39] But due to their differing degrees of commitment they will do so at different points in time. For General Discipline to obtain it is therefore required that $b(F_1^I) = \{M_1^D\}$, which yields the following condition analogous to (23)

$$b \sum_{j=1}^{J} f_j r_j^F + a \sum_{j=1}^{J} (r^M - f_j r_j^F) > d r^M. \tag{39}$$

Rearranging and substituting in the specific payoffs yields (18). □

[39] This will not necessarily be the case under $R > 0$ but the conclusions will be unchanged.