Inflation Nutters?
Modelling the Flexibility of Inflation Targeting

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Abstract

Opponents of explicit inflation targeting (including ex-Chairman Greenspan) have argued that a commitment to a numerical inflation target reduces monetary policy's flexibility, and is hence likely to increase output volatility. Our paper demonstrates that this claim may fail to account for the anchoring effect of explicit targets on expectations and wages found in the data by several empirical studies. This is done in a novel, asynchronous game theoretic framework that incorporates the concept of 'economically rational expectations' by *endogenizing* the frequency of the private sector's and the central bank's moves. We show that under an explicit long-term inflation target private agents may find it optimal to look-through shocks and alter expectations and wages only *infrequently* - to minimize the cost of processing information and/or wage negotiations. Due to such rational inattention, wages and expectations will be anchored at the target level. This is shown to make the policymaker's interest rate instrument more effective in stabilization, giving it greater leverage over the real rate. This implies that an explicit inflation target improves the variability tradeoff, ie it shifts the policy frontier inwards and makes both inflation and output less variable in equilibrium. Our analysis thus adds another dimension to the rule vs discretion debate by showing that a long-run rule may be compatible with (and in fact enhance the effectiveness of) short-run discretion. We conclude by arguing that our results are consistent with existing empirical evidence.

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'The extent to which inflation targeting regimes impair central bank flexibility is a matter of professional dispute. There is probably no way that this disagreement can be settled in the present state of economic knowledge' McCallum (2003)

'Inflation targeting, even without imposing a rigid rule, would unduly reduce the flexibility of the Fed to respond to new economic developments in an uncertain world.' Rudebusch and Walsh (1998)

'The argument that inflation targeting might increase output fluctuations can be turned on its head. I would argue that inflation targeting can actually make it easier to reduce output fluctuations and probably has done so. First, the presence of an inflation target provides an effective nominal anchor... ' Mishkin (2004)

1. Introduction

The 1990s was a decade of central banking reform. Most significantly, a number of countries adopted a regime known as inflation targeting (thereafter IT).[2] This paper focuses specifically on one advantage and one conflicting disadvantage of IT identified in the literature associated with the regime's defining feature - an *explicit* commitment to a numerical inflation target.

On one hand, it has been argued that legislated numerical targets are beneficial in stabilization as they better *anchor* inflation expectations and wages (eg Mishkin (2004) above, or Bernanke (2003), Goodfriend (2003), McCallum (2003), Mishkin (2004), Lacker (2005). On the other hand, it has been believed that such a commitment constrains the policymaker's *stabilization flexibility* (eg Rudebusch and Walsh (1998) above, or Greenspan (2003), Kohn (2003), Friedman (2004)), which may lead to higher output volatility and interfere with monetary policy's dual mandate.[3] Our paper contributes to the IT debate by (i) explicitly modelling both the anchoring and the flexibility channels, and (ii) establishing a link between them that may be crucial in assessing the regime's desirability.

Our analysis demonstrates that the inflexibility concerns of IT sceptics indeed seem what Woodford coined '*traditional prejudice of central bankers*'. It is shown that explicit long-run targets are not inflexible in output stabilization per se, ie inflation targeters are not necessarily 'inflation nutters' (King (1997)). The model shows that explicit inflation targets in fact *decrease* the variability of output under reasonable circumstances, due to their anchoring effect.

This is done using a novel game theoretic framework developed in Libich and Stehlík (2007), which is a simple generalization of alternating move games (Maskin and Tirole (1988)). Specifically, the players take actions at certain constant frequencies. In contrast to the commonly used rational expectations solution or a repeated game setting, this framework has several advantages.

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[2] For extensive treatments of IT see Bernanke et al. (1999), Svensson (1999), Blejer et al. (2000), Mishkin and Schmidt-Hebbel (2001), Truman (2003), or Bernanke and Woodford (2005).

[3] The meaning of 'stabilization flexibility' has not been precisely defined in the literature. We will use it in the sense of Bernanke (2003) as the ability '*to choose the best policies in the future*' in terms of inflation and output stabilization.

First, unlike these standard setups that are static (expectations and the policy instrument are always adjusted simultaneously), our framework allows for a more dynamic interaction between the policymaker and the public that combines synchronized and asynchronous moves. Second, unlike these standard setups in which gathering/processing information, updating expectations, or renegotiating wages is commonly costless, our framework incorporates some costs of these activities. Third, the frequency of the players' decisions can be made endogenous, ie optimally selected based on cost-benefit calculations.[4] As a consequence, actions may be infrequent due to the costs, in line with the increasingly popular concepts of 'economically rational expectations' (Feige and Pearce (1976)) and 'rational inattention' (Sims (2003) and Reis (2006)).

Motivation and Intuition. The inflexibility view of IT sceptics seems to be grounded in the following simple intuition. Let the policymaker's objective function be as follows: $U = -\alpha var(x) - var(\pi)$, where x, π, and $\alpha \geq 0$ denote the output gap, inflation, and their relative weight. A common way to think about IT is a lower α, which Rogoff (1985) coined a conservative central banker. It is straightforward to show that in the presence of aggregate supply shocks output volatility is indeed decreasing in α.

Is it however true that explicit IT implies stricter IT (lower α)? The answer is clearly affirmative if the inflation target is specified as a short-run objective that must be achieved at every point in time, since this would imply an inflation nutter with $\alpha = 0$. In contrast, a number of academics and central bankers have argued that the answer is negative if the inflation target is specified as a long-term objective (with the horizon being indefinite or the business cycle as in most industrial IT countries, see Mishkin and Schmidt-Hebbel (2001)).[5] This is because shocks have a zero mean so they do not affect the steady-state (average) levels of inflation and output. It then follows that such explicit long-run IT does not imply stricter IT, ie it does not necessarily affect the parameter α, the stabilization flexibility, and the volatility of the targeted variables.[6]

We join the latter body of work but go a step further. Our theoretical analysis first formalizes the anchoring effect of IT that has been reported by empirical studies, eg Gürkaynak et al. (2009), Beechey et al. (2008), Gürkaynak et al. (2005), Levin et al. (2004) or Kuttner and Posen (1999). It then shows that, due to this effect, the policy's flexibility may in fact increase (and output volatility decrease) under explicit IT, contrary to what IT opponents conjecture.

This parallels the finding of Orphanides and Williams (2005), and echoes the arguments of Bernanke (2003), Goodfriend (2003), and Mishkin (2004) that the extra

[4]This is called for by Tobin (1982): *'Some decisions by economic agents are reconsidered daily or hourly, while others are reviewed at intervals of a year or longer... It would be desirable in principle to allow for differences among variables in frequencies of change and even to make these frequencies endogenous'.*

[5]To document, Svensson (2009) notes that: *'Previously, flexible inflation targeting has often been described as having a fixed horizon, such as two years, at which the inflation target should be achieved. However, as is now generally understood, under optimal stabilization of inflation and the real economy there is no such fixed horizon at which inflation goes to target or resource utilization goes to normal.'*

[6]The policy responses to the global financial crisis demonstrate this also. Explicit inflation targeters responded vigorously to the situation willing to deviate from their inflation targets in order to stabilize the financial sector and the real economy.

credibility gained by IT enables central banks to reduce the interest rate more aggressively in response to shocks without 'upsetting' inflation expectations. Our finding is similar in that an explicit long-run inflation target better anchors the public's behaviour, which makes the policymaker's interest rate instrument more effective in stabilization. We however show that such anchoring leads to an improved variability tradeoff, and therefore a *less* aggressive interest rate response is required in equilibrium to achieve lower volatility of both output and inflation. Since interest rate volatility is disliked (see eg Woodford (1999)), this constitutes an additional advantage of explicit long-run IT.

Framework. We adopt the New Keynesian setup of Clarida et al. (1999), but extend it in terms of (i) the number of the players' instruments, (ii) the timing of the players' actions, and (iii) the costs associated with the frequency of actions.[7] First, each player has two 'instruments'. The public, player p, forms inflation expectations, π^e, and sets wage inflation, w. The policymaker, player g, chooses the short-run (SR) interest rate, i, and the level of the long-run (LR) inflation target, π^T (whose horizon is indefinite as implied by Svensson's quote above). In addition, each player has two other choice variables. The players optimally select the frequency with which the levels of these instruments can be reconsidered throughout the game.

This frequency choice is a one-off decision that happens at the beginning of the game. The moves that get set repeatedly then proceed as follows (see Figure 1). In the first period, observing the chosen frequencies, there is a simultaneous initial move of all instruments. This is assumed for maximum comparability with the conventional synchronized move settings. After that, each instrument $m \in \{\pi^e, w, \pi^T, i\}$ can be adjusted every $r_m \in \mathbb{N}$ periods. While both the public and the policymaker are forward looking, they may rationally choose to move infrequently - a high r_m. This is either to minimize the associated cost, $C_m(r_m)$, where $\triangle C_m/\triangle r_m \geq 0$, or to commit to their actions.[8]

Interpretation. We will refer to r_e as *expectations anchorness* and to r_w as *wage anchorness (or rigidity).* This interpretation is natural - the less frequently expectations and wages can be altered the more anchored (rigid) they are. We will refer to r_T as the degree of *explicitness of IT.* This is because arguably the more explicitly a long-run inflation target is stated in the central banking legislation or Statutes, the less frequently the target can be altered or the less likely it is.[9]

As a real world example of r_T, the 1989 Reserve Bank of New Zealand Act states that the inflation target may only be changed in a Policy Target Agreement (PTA) between the Minister of Finance and the Governor, and this can only be done on pre-specified regular occasions (eg when a new Governor is appointed). Such explicit arrangement implies a high value of r_T - since December 1990 the PTA was only 'reconsidered' five times, ie roughly every three years (and on three of these occasions the target was slightly

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[7]In order to do so effectively we make some simplifications to the setting of Clarida et al. (1999).

[8]We utilize throughout the standard definition of forward differences due to the discreteness of r_m: $\triangle f = f(.+1) - f(.)$.

[9]It is important to realize that our deterministic explicitness/anchorness ala Taylor (1979) can be reinterpreted as probabilistic in the spirit of Calvo (1983). The reader can think of a certain probability independent across time, $\theta_m = [0, 1)$, that instrument m cannot be reconsidered in a given period. Then the average/expected length of time between each reconsideration is $\frac{1}{(1-\theta_m)}$, which is equivalent to our r_m.

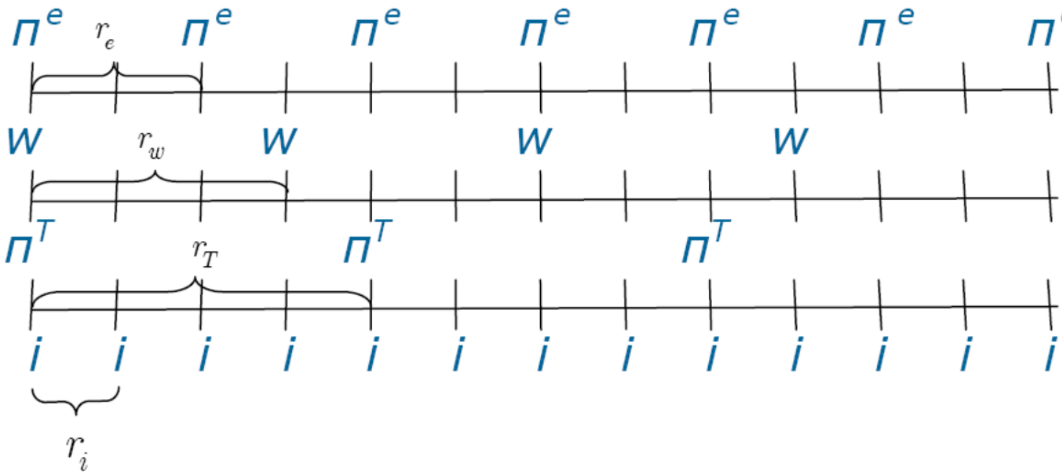

FIGURE 1. An asynchronous game: an example of the timing with $r_e = 2, r_w = 3, r_T = 4, r_i = 1$.

altered). In contrast, central banks without a legislated numerical target can arguably alter their inflation objective more readily.[10] Allowing IT explicitness to have various degrees $r_T \in \mathbb{N}$ rather than treating is as a binomial variable (targeters vs non-targeters) addresses Gertler's (2003) criticism of the existing literature.

It will become apparent that r_T can also be interpreted as a measure of LR commitment due to its role in tying the policymaker's hands in regards to average inflation. As such, the explicitness of the target effectively plays a role of an additional policy instrument. However, the setup makes it apparent that this commitment concept is very different from the standard pre-commitment solution (timeless perspective) popularized by Woodford (1999) and Clarida et al. (1999) in which $r_T = r_i = 1$ is implicitly assumed. We discuss the links between the two concepts in Section 6.

Results. Using the above notation, let us summarize the main objective of the paper and its results. Our goals is to examine the interaction between the two instruments of the policymaker, namely the effect of the explicitness of LR inflation target, r_T, on the flexibility of the SR interest rate, i, and the resulting stabilization outcomes, $var(x^*)$ and $var(\pi^*)$. In the terminology of Kydland and Prescott (1977), we study the impact of a LR rule on SR discretion. In doing so we establish an indirect link between them through the anchorness of the public's behaviour, r_w and r_e.

Our main result is composed of two findings. In the first step, called the 'LR game', it is shown that if (and only if) the target is *sufficiently* explicit, then the equilibrium anchorness levels r_w and r_e are increasing in the degree of the IT's explicitness, $\frac{\triangle r^*_{w,e}}{\triangle r_T} > 0$. This implies that the public will then optimally look through shocks (see Brash (2002)) to save some cost of updating expectations and renegotiating wages. Wages and expectations will therefore be anchored at the inflation target level. As a second step, the

[10]Nevertheless, an absence of a legislated inflation target does not necessarily imply $r_T = 1$. A central bank pursuing an inflation target *implicitly* (see eg Goodfriend (2003) for the US, and Bernanke et al. (1999) for the Bundesbank and the Swiss National Bank in the 1980s) can be described by some $r_T > 1$.

'SR game' shows that the variability of inflation and output in equilibrium is decreasing in the degree of anchorness, $\frac{\triangle var(\pi^*, x^*)}{\triangle r_{w,e}} < 0$. This is because the public's rational inattention to shocks gives the policymaker greater leverage over the real interest rate and the economy. Combining these two findings implies that the more explicit IT, the less variability in both goal variables as well as in the policy instrument, $\frac{\triangle var(\pi^*, x^*, i^*)}{\triangle r_T} < 0$.

The paper further finds that there exists substitutability between the explicitness of IT and the regime's strictness (central bank conservatism/independence) in securing the policy credibility. This may explain why inflation targets have been implemented more explicitly by countries that had lacked central bank independence in the late 1980s such as New Zealand, Canada, UK, and Australia, rather than those with an independent central bank such as the US, Germany and Switzerland.

In the working paper version of this article we extensively discuss the existing empirical evidence related to our results, both the LR and the SR, and show that there exists fair support for all our findings. Due to the space constraint, the presented version only briefly reports these papers.

Let us point out one important issue our analysis leaves for future research. While our paper highlights the benefits of an explicit inflation target, it does not examine the exact specification of the inflation objective. Real world targets have been specified in terms of consumer prices, but the build up of the global financial crisis of 2007-9 brings back to the fore the question of whether central banks should respond to a broader measure of inflation also including various asset prices.[11] Therefore, the term inflation in our analysis should be interpreted *broadly* as potentially including more than just consumer prices - more research is required to shed light on this issue.

The rest of the paper is structured as follows. Section 2 describes the model. Section 3 first presents the standard repeated game and then introduces our generalized asynchronous framework in which moves may be endogenously infrequent. Section 4 examines the LR outcomes whereas Section 5 considers the SR outcomes. Section 6 discusses the robustness of the results and several extensions. Section 7 summarizes and concludes.

2. The Model

Throughout, both the public and the monetary policymaker (the central bank or the government) are assumed to be rational, have common knowledge of rationality, and complete information about the economy and the structure of the game. We will further abstract from the players' discounting for simplicity. These assumptions are made in order to better focus on the costs associated with the frequency of the players' actions, $C_m(r_m)$. It will become clear that the intuition of our results applies even if these assumptions are relaxed.

In the standard New Keynesian model of Clarida et al. (1999) the economy is described by two equations, namely a Phillips curve and an IS curve.[12] We extend both to feature w, the rate of change of nominal wages (wage inflation) as in Rogoff (1985). Using the

[11] The existing pre-crisis literature commonly answered this question in the negative, eg Bernanke and Gertler (2001).

[12] How these (type of) equations arise from a micro-founded model featuring optimizing households and firms see for example Woodford (2003).

notation of Section 1 we have

$$\pi_t = \lambda x_t + \gamma \pi_t^e + (1-\gamma) w_t + u_t, \tag{1}$$

$$x_t = -\kappa(i_t - \pi_t^e) - \varphi(i_t - w_t) + q_t, \tag{2}$$

where $t \in \mathbb{N}$ denotes time and $\lambda > 0$, $\gamma = [0,1]$, $\kappa \geq 0$, $\varphi \geq 0$ are parameters.[13] The disturbances follow the usual AR1 processes

$$u_t = \rho u_{t-1} + \hat{u}_t \quad \text{and} \quad q_t = \mu q_{t-1} + \hat{q}_t, \tag{3}$$

where $\rho \in [0,1)$, $\mu \in [0,1)$, $\hat{u} \sim \text{iid}(0, \sigma_u^2)$ and $\hat{q} \sim \text{iid}(0, \sigma_q^2)$. The policymaker's period preferences are

$$U_t^g = -\alpha(x_t - x^T)^2 - (\pi_t - \pi^O)^2 - C_T - C_i, \tag{4}$$

where the inflation target is at the socially optimal level (which we throughout normalize to zero, $\pi^O = 0$). The output gap target can however be positive, negative or zero, $x^T \in \mathbb{R}$.[14] As discussed above, α expresses the reciprocal of the degree of *conservatism/strictness of IT* (the literature building on Rogoff (1985) interprets it as a measure of central bank independence). Further, C_T is an *IT implementation cost* and C_i is a monetary policy committee *meeting cost* (both will be discussed below). The public's period utility function is the following

$$U_t^p = -\beta(\pi_t - \pi_t^e)^2 - (\pi_t - w_t)^2 - C_\pi - C_w - C_e, \tag{5}$$

where C_π is an *inflation cost*, C_w is a *wage bargaining cost*, and C_e is an *expectation updating cost* (all will be postulated below). The intuition of the first three components is conventional, and equivalent to rational expectations, see Backus and Driffill (1985). An inflation averse public attempts to correctly expect the inflation rate in order to set wages at the market clearing level (for a justification see Canzoneri (1985)). The additional C_w and C_e elements underlie the body of work on rational inattention (Sims (2003) and Reis (2006)), and will enable us to formalize the concept of 'economically rational expectations', in which the players' frequency of updating expectations is a result of cost-benefit calculations.

3. The Game Theoretic Setup

3.1. **Standard Timing: Frequent and Synchronized Moves.** The policy has been, at least since Barro and Gordon (1983), commonly studied as an (infinitely) repeated game. Under discretion, as well as under pre-commitment (timeless perspective), players' instruments are adjusted *simultaneously* at *each* period t, ie $r_m = 1, \forall m$. The same is implicitly assumed in conventional rational expectations models.

[13] The timing of π_t^e will be specified (endogeneously) in Section 3.2. It will nevertheless be discussed in Section 6 that the intuition of our findings holds for various specifications of expectations π_t^e including the common $E_t \pi_{t+1}$.

[14] $x_T \neq 0$ can be due to mismeasurement of potential output (eg Orphanides (2001)), market imperfections (eg Kydland and Prescott (1977), Barro and Gordon (1983)), political economy reasons (eg Faust and Svensson (2001)), a shortcut way to reflect asymmetry in the policymaker's preferences (as in eg Cukierman and Gerlach (2003), Ruge-Murcia (2004)), or to represent spillovers from excessive fiscal policy (in the spirit of Sargent and Wallace (1981)).

Under this special case our model yields outcomes analogous to the New Keynesian setting of Clarida et al. (1999). To see this, set up the Lagrangian, disregard the costs C in (4)-(5), and impose rational expectations to obtain the familiar targeting rule under discretion

$$\pi_t = -\frac{\alpha}{\lambda}(x_t - x^T). \tag{6}$$

Substituting (6) into the Phillips curve yields the values of inflation and the output gap in equilibrium (denoted by asterisk throughout)

$$\pi_t^* = \frac{\alpha}{\lambda^2}(u_t + \lambda x^T) \text{ and } x_t^* = -\frac{1}{\lambda}u_t, \tag{7}$$

As is standard, the supply shock in (7) does not affect the steady-state values due to its zero mean. This implies independence of LR levels from SR disturbances, ie the mutual consistency of the two instruments of the policymaker. Formally, denoting all LR variables by a bar and using $\bar{u} = 0$ with (7) yields the following LR equilibrium levels

$$\bar{\pi}^* = \frac{\alpha}{\lambda}x^T \text{ and } \bar{x}^* = 0. \tag{8}$$

This independence means that the policymaker can be, in some period t in which a supply shock occurs, *consistently* committed to the optimal LR inflation target, π^O, but choose a different level of inflation, $\pi_t^* \neq \pi^O$, that maximizes its objective function according to (6)-(7). This feature is obviously not specific to our model; it is present in most settings used in monetary analysis including Clarida et al. (1999).

3.2. **Generalized Timing: Possibly Infrequent and Asynchronous Moves.** Our framework is a generalization of the asynchronous (alternating) move setups of Maskin and Tirole (1988) and Lagunoff and Matsui (1997), that follows the recommendation of Cho and Matsui (2005): *'[a]lthough the alternating move games capture the essence of asynchronous decision making, we need to investigate a more general form of such processes... '*. In doing so the framework draws on the intuition of games with endogenous timing, eg Bhaskar (2002).

Game Theoretic Assumptions: The version of the framework used here adopts the main features of a standard repeated game for comparability. First, time is discrete. Second, all r_m values are, as defined in the introduction, deterministic in the spirit of Taylor (1979). Third, all r_m's are constant and observable by the players. Third, the opponent's preceding periods' moves can be observed (ie games of perfect monitoring). Fourth, the game starts with a simultaneous move of all actions. We postulate the following timing that we find most realistic in the monetary policy context.

(1) The policymaker selects r_T and r_i. Observing these, the public chooses r_e and r_w. All these choices apply throughout the whole game.
(2) At the beginning of every period t there is a realization of shocks, u_t and q_t.
(3) In period 1 observing all r_m as well as u_1 and q_1, the players simultaneously set the levels of $\{\pi^e, w, \pi^T, i\}$.

(4) The $\{\pi^e, w, \pi^T, i\}$ levels can then be reset every $\{r_e, r_w, r_T, r_i\}$ periods respectively, observing current and past shocks.[15]

A strategy for a certain player is a function that, for all histories, assigns a probability distribution to the players' action space. As common in macroeconomics, in this paper we will restrict our attention to pure strategies. A strategy of a player is then a vector that, for all histories, specifies the player's play in every node. The asynchronous game will commonly have multiple Nash equilibria. To select among these we will use a standard equilibrium refinement, subgame perfection, that eliminates non-credible threats. Subgame perfect Nash equilibrium (SPNE) is a strategy vector that forms a Nash equilibrium after any history. Following the macroeconomic literature, we will only focus on Markov perfect equilibria, see eg Maskin and Tirole (2001).

Macroeconomic Assumptions: In terms of $C_i(r_i)$ the literature has not put forward any reasons for a non-zero value – this is because the cost of more frequent monetary policy committee meetings seems trivial relative to the macroeconomic consequences. Therefore, we will assume $C_i(r_i) = 0, \forall r_i$. Similarly, we will set $C_T(r_T) = 0, \forall r_T$. This is because: (i) the possible *inflexibility* cost of explicit IT is formally examined in this paper, and (ii) other related costs (such as an implementation cost) are arguably negligible relative to the stabilization outcomes.[16]

In contrast, the fact that there exist non-trivial inflation and wage bargaining costs, $C_\pi > 0, C_w > 0$, is uncontroversial. In terms of the former see eg Romer and Romer (1997) or McCallum and Nelson (2004). In terms of the latter Mankiw and Reis (2002) discuss the existence of costs related to *'changing wage contracts and information-gathering, decision making, negotiation and communication'* (see also the literature on wage rigidity initiated by Fischer (1977) and Taylor (1979) and its empirical evidence, eg Bewley (2002)).

We therefore assume the following costs in (5). The inflation cost is a fixed per-period cost that incurs if LR inflation differs from the optimal LR level, ie

$$C_\pi = \begin{cases} c_\pi > 0 & \text{if} \quad \bar{\pi} \neq \bar{\pi}^O, \\ 0 & \text{if} \quad \bar{\pi} = \bar{\pi}^O. \end{cases} \tag{9}$$

The wage bargaining cost is a per-period fee increasing in the number of wage negotiations, $\frac{\triangle C_w}{\triangle r_w} < 0, \forall r_w$. In order to be able to derive closed-form analytical solutions, and to make the game theoretic analysis more transparent, several simplifying assumptions will be made. First, we use the following functional form for the wage bargaining cost

$$C_w = \frac{c_w}{r_w}, \tag{10}$$

where $c_w > 0$. Second, as a matter of experimental control we will separate the effects of anchored (rigid) wages and anchored (sticky) expectations. This is because π^e is

[15] Such full information is often assumed in the New Keynesian framework. It will make it possible to attribute the public's inattention to not-processing the available information rather than to not-possessing information. Therefore, C_e has been labelled as the cost of updating expectations rather than cost of acquiring information. For analyses of a situation in which the policymaker has private information about shocks see the 'transparency' literature initiated by Cukierman and Meltzer (1986).

[16] It will become evident that even if we allow for these costs to be positive, the intuition of our findings will be unchanged as long as the costs are below a certain threshold, $0 < C_T \leq \tilde{C}_T$ and $0 < C_i \leq \tilde{C}_i$.

used in setting w and these two actions are therefore interconnected. As our companion paper Libich (2009) formally models anchorness of expectations, we will here restrict our attention to wages.[17] Nevertheless, Section 6 explains why the intuition and impacts of the two effects are analogous.

In focusing on wage anchorness we will assume $C_e(r_e) = 0, \forall r_e$, and in line with this $\kappa = \gamma = 0$. It then follows from (5), in combination with $\beta > 0$ and (7), that the public will choose to update expectations every period, $r_e^* = 1, \forall r_T, r_i, r_w$. Intuitively, the supply shock may occur every t, and it affects π_t^* (see (7)), which the public attempts to correctly expect (see (5)). This in turn, combined with the assumed full information, implies $\pi_t^{e*} = \pi_t, \forall t$, and has two advantages. It coincides with the assumption underlying the standard rational expectation solution. Further, the analysis will be simplified as the public will be setting wages using the 'correct' inflation expectations. Nevertheless, we will in Section 6 also discuss the cases $C_e, \kappa, \gamma > 0$ (and hence $r_e^* > 1$).

Two Stage Analysis. Due to the same expositional considerations, it is beneficial to present the results in two separate (but interconnected) parts: the 'LR game' and the 'SR game'. The LR game will focus on the trend outcomes, primarily on setting the inflation target and trend wage growth, and will therefore consider the game under the assumption of no shocks. This part will derive the LR conditions for the inflation target to be credible, and for it to have an anchoring effect. In doing so it will communicate the intuition of the asynchronous game and its solution.

The SR game will then feature the full model including shocks and study the resulting deviations from the reported LR outcomes. This part will derive an additional SR condition for anchoring, and show its stabilization effects. The interplay of the LR and SR perspectives is important - it will become apparent that while the anchoring effect is a LR phenomenon, its macroeconomic impact extends to SR stabilization as well.

4. The LR Game

Because shocks do not affect the LR (average) levels of the variables, see (7), we will in this section disregard them (as well as the related r_i choice) by imposing $u_t = q_t = 0, \forall t$. In the absence of shocks the above introduced asynchronous LR game has a *stage game* that (i) is itself a dynamic game, and (ii) lasts M periods where $M \in \mathbb{N}$ is the 'least common multiple' of all r_m. To demonstrate, in the example of Figure 1 we have $M(r_T = 4, r_w = 3, r_e = 2) = 12$.

Normal Form Game. In order to be able to present the game in the normal form we follow the game theoretic literature (eg Cho and Matsui (2005)) and restrict the LR game to two levels of π^T and w: one optimal, O, and one sub-optimal, S. Specifically, we select the inflation level and the output gap target level:[18]

$$\pi^T \in \{\pi^O = 0, \pi^S = \lambda x^T\} \ni w. \tag{11}$$

[17] Wage rigidity is an established concept with sufficient empirical support. Further, Levin et al. (2005) find that the performance of optimal policy is closely matched by a simple *wage stabilization rule* and stress *'the importance of additional research regarding the structure of labor markets and wage determination'*.

[18] The output target in (11) is normalized by λ to simplify the payoff functions. Under different O and S values the results of the LR game would differ quantitatively, but their qualitative nature would be intact, see Libich (2009) for a different truncation of the action sets.

Using this with (1), (4), and (5) we can derive the following payoff matrix of the standard *static* (one period) stage game, where the first payoff $\{a, b, c, d\}$ refers to the row player g and the second payoff to the column player p.[19]

(12)

		Public	
		w^O	w^S
Policymaker	π^O	$a = -\alpha; 0$	$b = -4\alpha; -1$
	π^S	$c = -\lambda^2; -(c_\pi + 1)$	$d = -(\alpha + \lambda^2); -c_\pi$

Due to the truncation of the LR action sets in (11), we will make three technical assumptions. First, in order for the players to always have a choice between two different levels, we need to exclude the case in which the O and S levels are the same by imposing $\pi^S = \lambda x^T \neq \pi^O$. Note however, that π^S can still be both greater or less than π^O.

Second, in order to still preserve the time inconsistency feature in our truncated game, ie for $c > a$ to hold, we only consider the cases $\alpha > \lambda^2$. This is however not very restrictive since the real world value of λ is small; for example, Rudebusch (2002) estimates it to be $\lambda = 0.13$. It is apparent that, in line with the intuition of Kydland and Prescott (1977), the game has a unique Nash equilibrium, (π^S, w^S), which is Pareto inferior to the efficient outcome (π^O, w^O). We will show that allowing for asynchronous moves may alter the outcomes of the game.

Third, we will throughout focus on the case of interest, $r_T > r_w$ - the equilibrium r_T^* will be later shown to satisfy this condition. In this region let us restrict our attention to the intuitive special case of $r_T = nr_w$, where $n \in \mathbb{N}$. This assumption reduces the degree of asynchronicity, and the length of the dynamic stage game to $M(r_T, r_w, r_e) = r_T$ periods (one move of the policymaker and n wage moves of the public). Nevertheless, Libich and Stehlík (2007) demonstrate that this special case $n \in \mathbb{N}$ is representative of the more asynchronous cases $n \notin \mathbb{N}$ in which the dynamic stage game can be up to $M(r_T, r_w, r_e) = r_T r_w r_e$ long. We will refer to the periods in which wages can be renegotiated as *Cournot* periods and those in which wages cannot be adjusted as *Stackelberg* periods.

Horizon and Repetition. While the dynamic stage game can be repeated any (finite of infinite) number of times, we will abstract from its repetition when examining the LR game. This can be done, without loss of generality, because (i) in the LR game there are no shocks, and (ii) we will be interested in conditions under which the sole *Pareto-efficient* outcome (π^O, w^O) *uniquely* obtains on the equilibrium path of the dynamic stage game. Then repeating the stage game, and allowing for reputation building of some form, would not affect the reported equilibrium. Intuitively, if the dynamic stage game has a unique SPNE that is efficient, then the effective minimax values of the repeated game will be equivalent to those of the dynamic stage game - since these cannot be improved upon. Put differently, since the outcome lies on the Pareto frontier the set of Pareto superior payoffs is empty.[20]

[19] For illustration we set $c_w \to 0$ and $(x^T)^2 = 1$ in (12).

[20] For the fact that the Folk theorem may not apply in asynchronous games, which is the case here, see eg Takahashi and Wen (2003).

Let us define the following concept drawing upon the intuition of the literature, eg Faust and Svensson (2001) and Demertzis et al. (2008).

Definition 1. *An inflation target will be called* ***credible*** *if the public (i) expects this inflation level on average in equilibrium,* $\bar{\pi}^{e*} = \pi^O$*, and therefore, (ii) optimally sets trend wage growth at this level,* $\bar{w}^* = w^O = \pi^O$*. If either (i) or (ii) are not satisfied, ie if expectations and/or wages differ from the inflation target on average, the target will be called to* ***lack credibility****.*

Note that as the inflation target is a LR objective, its credibility depends on expectations of average inflation and average wages. The following definition relates to our solution concept, subgame perfection.

Definition 2. *Any SPNE that has, throughout its equilibrium path, both players in the LR game playing the optimal O average levels will be called* ***Ramsey****.*

We can now propose two sets of results for the LR game. Their order is implied by the backwards induction solution of the game used. The first proposition reports findings about the effect of r_T on average inflation, wages, and the inflation target's credibility (treating both r_T and r_w as exogenous). The second focuses on the relationship between r_T and r_w and reports the anchoring effect (treating r_T as exogenous and r_w as endogenous). Section 5 will show that all these results obtain even in the SR game in the presence of stochastic disturbances. It will then report results relating to the policymaker's optimal IT explicitness decision (treating both r_T and r_w as endogenous).

Proposition 1. *(i)* ***IT Credibility****: The* π^O *level of the inflation target is credible if and only if the target is sufficiently explicit. Only then the policymaker behaves 'as if' he targets the natural rate of output even if* $x^T \neq 0$*, and such behaviour is credible.*
(ii) ***IT Substitutability****: Explicitness of IT and strictness of IT (conservatism) are partial substitutes in achieving the target's credibility.*

Proof. The proposition states that (i) iff the target's explicitness is above some threshold, then the O levels will obtain throughout the LR game on average: $w_t^{O*} = \pi_t^{e*} = \pi_t^{O*}, \forall t, \alpha, x^T$. Put differently, the game has a unique SPNE that is of the Ramsey type. Appendix A derives a *necessary* credibility condition

$$r_T \geq \frac{\alpha}{\lambda^2} r_w, \tag{13}$$

as well as a *sufficient* credibility condition

$$r_T > \tilde{r}_T = \frac{3\alpha}{\lambda^2} r_w. \tag{14}$$

These conditions also prove claim (ii) by showing that the threshold levels are a decreasing function of the policy's conservatism/strictness (increasing in α). □

Interestingly, unlike in the standard repeated game of the Barro and Gordon (1983) type, the optimal inflation target may be time consistent and credible even if $\alpha > 0$ and $x^T \neq 0$; and this is true even in the worst case scenario of a finite game without reputation building. To achieve this, the target must be sufficiently explicit. Intuitively, under $r_T > r_w$ the public gets to re-adjust wages *after* it has observed the level of inflation (with the exception of all Cournot periods in which the players move simultaneously). Therefore, if the policymaker plays the S level then the public will get to 'punish' him with S level wages and the inefficient Nash.[21]

The proof in Appendix A shows that if $r_T > \tilde{r}_T$, then this punishment is long enough to offset the possible output benefit of the policymaker from surprise inflating/deflating, and eliminate his temptation to do so even if $x^T \neq 0$. Knowing this, the public sets expectations and wages at the O level, and π^O then becomes credible. Put differently, the policymaker's explicit IT provides a LR commitment by sufficiently tying the policymaker's hands in terms of *average* inflation.

A number of authors, eg McCallum (1997) and Blinder (1997), argued that a simple recognition of the fact that $x^T \neq 0$ leads to undesirable outcomes is sufficient to constrain the policymaker's behaviour, ie he then acts 'as if' $x^T = 0$. They do not however formally show how such behaviour will be achieved in equilibrium, and under what circumstances it will be credible in the eyes of the public. Our analysis does so by deriving a sufficient degree of IT's explicitness, $\tilde{r}_T$, that ensures credibility throughout for all $x^T \in \mathbb{R}$.

This implies that a mere announcement of a preferred average inflation level, without it being legislated, may not provide a sufficiently strong commitment. This is because it is easier to be reneged upon in the next period. Furthermore, the *observational equivalence* of Proposition 1 suggests that caution should be exercised in concluding that a track record of low inflation necessarily indicates a conservative and responsible policymaker, $x^T = 0$, without any temptation to over-stimulate output.

Note that the explicitness of the target is an alternative avenue to ensure credibility - independent of the effect of reputation and past performance (that are examined in a large body of the literature including Hughes Hallett and Libich (2007)). Nevertheless, it needs to be stressed that a legislated inflation target does not imply credibility per se, it works through current and future performance. By providing the right incentives to the central bank it ensures that inflation is (on average) on target, and this is why credibility is enhanced. In this sense it can therefore be argued that credibility cannot be legislated, it must be earned. Explicit IT simply helps in earning it by altering the incentives of the policymaker, along the lines of Walsh (1995).

Let us briefly report two auxiliary results implied by our LR analysis.

Remark 1. *The substitutability result of Proposition 1(ii) offers an explanation for why countries with low degree of central bank (goal) independence have formulated their inflation targets more explicitly than those with a highly independent central bank.*

Libich (2008) presents empirical evidence for this claim using the degrees of accountability and transparency reported in established indices as proxies for the degree of IT explicitness. For example, it is shown that the correlation between transparency in Eijffinger and Geraats (2006) and goal-independence in Briault, Haldane and King (1997) is -0.86 ($t = -4.46$). Intuitively, countries lacking central bank independence in the first half of the 1980s, such as New Zealand, Canada, UK, Sweden and Australia, had

[21] Note that unlike in Barro and Gordon (1983), the punishment in the asynchronous game is not arbitrary but it is the public's optimal play, and its length is uniquely determined by the length of the wage contract.

a greater payoff from committing to an inflation target more explicitly than countries with a relatively independent central bank such as the US, Germany and Switzerland.

It should be noted that our explanation is in line with the result of Schaling and Nolan (1998), and the hypothesis of Briault, Haldane and King (1997): '*The negative correlation ... suggests that accountability and transparency may have served as (partial) substitutes for independence...*'.

Remark 2. *Initial conditions matter. After an inflationary/deflationary period a more explicit and/or stricter IT may be necessary to ensure the credibility of the optimal inflation target.*

Combining the necessary and sufficient conditions (13) and (14) implies that under $\frac{r_T}{r_w} \in [\frac{\alpha}{\lambda^2}, \frac{3\alpha}{\lambda^2}]$ there exist multiple SPNE, including both Ramsey and non-Ramsey SPNE.[22] The selected outcome is then likely to depend on the past, ie players will continue playing the previous SPNE. Therefore, certain degrees of IT's explicitness and strictness that may have been sufficient for credibility in a stable price level environment, may be insufficient in the aftermath of a prolonged inflationary or deflationary period. Note that this is true in our model even under a purely forward-looking public, ie not due to adaptive expectations.

This perhaps helps explain why institutional arrangements such as explicit IT and conservative/independent central bankers may not have been needed before the 1970s, but were imperative for credibility afterwards. Further, it points to a solution to the deflationary problem of Japan of the past two decades (that is in line with Svensson (2001)) – an appointment of an explicit inflation targeter. Similarly, a number of economists have recently suggested that a more explicit inflation objective would be beneficial in avoiding deflation as a consequence of the global financial crisis of 2007-9.

The next proposition examines the relationship between explicit IT and the equilibrium frequency of wage negotiations.

Proposition 2. ***Anchoring Effect (LR)****: If the inflation cost is sufficiently high relative to the wage bargaining cost, $c_\pi > \tilde{c}_\pi(c_w)$, then a sufficiently explicit inflation target, $r_T > \tilde{r}_T$, anchors wages. Specifically, wage anchorness is increasing in the degree of IT's explicitness.*

Proof. The proposition claims that $\frac{\triangle r_w^*}{\triangle r_T} > 0, \forall r_T > \tilde{r}_T$ and $c_\pi > \tilde{c}_\pi(c_w)$. Appendix B derives the latter threshold and shows that $\tilde{c}_\pi > c_w$ as well as $\tilde{c}_\pi \leq c_w$ under some circumstances. The Appendix then derives equilibrium wage anchorness to be, under the stated conditions,

$$r_w^* = \frac{\lambda^2 r_T}{3\alpha}. \tag{15}$$

The fact that r_w^* is an increasing function of r_T proves the claim. □

[22] This implies that equilibrium credibility in our framework is a jump function of the explicitness of the target. If equation (14) is satisfied then expectations and wages are always at the O level, and hence the target is always fully credible as per Definition 1. If $\frac{r_T}{r_w} \in [\frac{\alpha}{\lambda^2}, \frac{3\alpha}{\lambda^2}]$ then the target may or may not be credible, since expectations and wages may be at the O or S levels. If $r_T < \frac{\alpha r_w}{\lambda^2}$ then the target is never credible, since expectations and wages are surely at the S level in some periods. So it is often - but not always - true in our model that a more explicit target leads to an increase in credibility.

Intuitively, in its r_w choice the public faces a tradeoff between minimizing two costs: the wage bargaining cost (by selecting long contracts, high r_w) and the inflation cost (by selecting sufficiently short contracts, $r_w \leq r_w$, and ensuring π^{O*}). If $c_\pi > \tilde{c}_\pi(c_w)$ then the public settles for the latter, but chooses the highest possible r_w that ensures credibility of the optimal average inflation level with certainty. This is the level implied by (14).[23]

5. The SR Game

In this section there exist shocks as specified in (1)-(3). Let us first note that our results of the LR game carry over to this stochastic environment. In terms of Proposition 1, both claims still obtain under shocks since these have a zero mean and do not affect the LR inflation level (see (8)). It then follows that they affect neither LR expectations and wages, nor the credibility of the LR inflation target. In terms of Proposition 2, we will prove the anchoring effect to still exist in the presence of shocks (under one additional condition on the magnitude of the supply shock σ_u^2). To be able to focus on the SR deviations in this section we will start off by simply assuming that the sufficient credibility condition in (14), $r_T^* > \tilde{r}_T$, is satisfied and prove it ex-post.

Our first SR game result relates to the policymaker's optimal choice of r_i, the frequency of the policy meetings (ie potential interest rate decisions).

Proposition 3. ***Frequency of Policy Decisions:*** *The policymaker will find it optimal to have the ability to adjust the interest rate (and hence inflation and output) every period,* $r_i^* = 1$.

Proof. Under $r_T > \tilde{r}_T$ the optimal LR levels obtain, so the game is played 'as if' $x^T = 0$ - see Proposition 1(i). Therefore, the optimal targeting rule in (6) effectively becomes, $\forall x^T$

$$\pi_t = -\frac{\alpha}{\lambda} x_t. \tag{16}$$

The simplest way to prove that $r_i^* = 1$ is to note that under $r_w = 1$ the (π^*, x^*) combination is consistent with the optimal targeting rule in (16) in every period and under any circumstances, and show that under $r_i > 1$ there exist circumstances that lead to (π^*, x^*) deviating from (16), which we do in Appendix C. □

This result is intuitive. Since shocks occur every period, being able to respond to them promptly (the period they occur) clearly enhances the effectiveness of stabilization. The result seems consistent with the real world practice whereby central bankers commonly have regular monthly meetings. Their frequency coincides with major data releases and can hence be interpreted as $r_i = 1$. The next proposition reports the macroeconomic effect of anchored wages.

Proposition 4. ***Effect of Anchorness:*** *Assume that $\alpha \geq \tilde{\alpha}$, where $\tilde{\alpha}$ is some positive threshold level. Then for any persistence of supply shocks ρ, wage anchorness is beneficial*

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[23] Note that since there exist Ramsey type SPNE in the multiple region $\frac{r_T}{r_w} \in \left[\frac{\alpha}{\lambda^2}, \frac{3\alpha}{\lambda^2}\right)$, the public could perhaps choose a somewhat longer contract. But since the policymaker may still be tempted to surprise inflation, we pay attention to the sufficient rather than necessary conditions for credibility. Finally, realize that if the sufficient conditions of Proposition 2 are not satisfied then there may be no anchoring effect, ie our result obtains weakly but is never reversed.

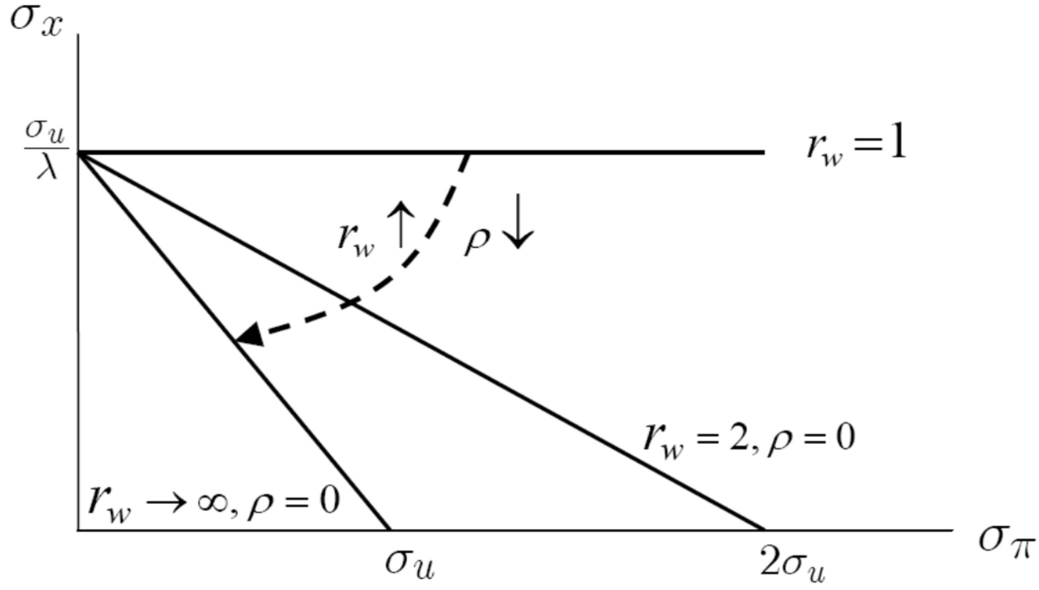

FIGURE 2. A schematic demonstration of the efficient policy frontier (Taylor curve) in Cournot periods. Higher r_w and lower ρ shift the frontier inwards.

in stabilization by increasing policy flexibility and hence decreasing the variability of both inflation and output in equilibrium. The persistence ρ has, for any r_w, the opposite effect.

Proof. The proposition claims that if $\alpha \geq \tilde{\alpha} > 0$ then $\frac{\triangle var(\pi^*, x^*)}{\triangle r_w} < 0, \forall r_w, \alpha, \rho$ and $\frac{\partial var(\pi^*, x^*)}{\partial \rho} > 0, \forall \rho$. Appendix D presents the proof and shows that $\tilde{\alpha}$ is, for realistic values of λ, very close to zero, ie the claim obtains for all 'reasonable' parameter values. □

We follow Clarida et al. (1999) and demonstrate the policy tradeoff by constructing the efficient policy frontier – the so-called Taylor curve. Figure 2 depicts the locus of points that characterize how equilibrium σ_π and σ_x under the optimal policy vary with r_w. It shows that the policy tradeoff in Cournot periods improves (the frontier shifts in) with an increasing value of wage anchorness r_w and decreasing persistence of supply shocks ρ.

Recall that our definition of stabilization flexibility follows Bernanke (2003): it is the ability 'to choose the best policies in the future'. We interpreted it to refer to inflation and output variability, since these are the two objectives in the policymaker's utility function. Let us now specify the conditions under which the anchoring affect, established in Proposition 2, will be present even in a stochastic economy.

Proposition 5. ***Anchoring Effect (SR):*** *Assume that the wage bargaining cost is sufficiently small relative to the inflation cost, $c_\pi > \tilde{c}_\pi(c_w)$, but sufficiently large relative to the magnitude of supply shocks, $c_w \geq \tilde{c}_w(\sigma_u^2)$, where $\frac{\partial \tilde{c}_w}{\partial \sigma_u^2} > 0$. Then a sufficiently explicit inflation target, $r_T > \tilde{r}_T$, anchors wages (for all $\alpha \geq \tilde{\alpha}$).*

Proof. It is shown in Appendix E that under the stated conditions we have $\frac{\triangle r_w^*}{\triangle r_T} > 0$. $\square$

Intuitively, the public's choice of r_w under shocks features the same tradeoff as in the LR game. A short wage contract, low r_w, is costly (implies higher C_w), but ensures better alignment of wages with inflation which is even more important in the SR game due to shocks. Clearly, under very large supply shocks, high σ_u^2, the public will choose to have fully flexible wages to respond promptly, $r_w^* = 1$, in which case wages are not anchored. In contrast, for sufficiently small σ_u^2 (relative to c_w) longer term contracts will be optimal. In such case r_w^* is, even in the presence of shocks, a monotonically increasing function of r_T, ie the anchoring effect occurs.

Proposition 6. ***Optimal IT Explicitness:*** *If $c_\pi > \tilde{c}_\pi(c_w)$ then it is optimal for the policymaker to make its LR inflation target sufficiently explicit.*

Proof. The proposition claims that $r_T^* > \tilde{r}_T$ for all $\sigma_u^2 \geq 0$. Recall from the LR game that the policymaker's motive for a sufficiently explicit IT commitment is to ensure the target's credibility. This motive still exists in the SR game, and is unaffected by the existence of disturbances, since the inflation target is a LR objective. In addition to this motive the policymaker wants to ensure the beneficial anchoring effect (Proposition 4). This, together with $C_T = 0$ and (14), implies $r_T^* > \tilde{r}_T$, and completes the proof. $\square$

The analysis implies that, under the stated circumstances, the policymaker should commit as explicitly as possible, $r_T^* \to \infty$, to maximize the size of the anchoring effect. We can now combine the above findings to formulate the main result of the paper.

Proposition 7. ***Effect of IT Explicitness:*** *Assume $c_\pi > \tilde{c}_\pi(c_w), c_w \geq \tilde{c}_w(\sigma_u^2)$ and $\alpha \geq \tilde{\alpha}$. Then a sufficiently explicit IT, $r_T > \tilde{r}_T$, increases the policymaker's stabilization flexibility, and hence reduces the volatility of output, inflation and the interest rate.*

Proof. It is shown in Appendix F that under the reported circumstances it is true that $\frac{\triangle var(x^*,\pi^*,i^*)}{\triangle r_T} < 0, \forall r_T$. $\square$

Intuitively, improved stabilization will be achieved by a less aggressive interest rate response in both Cournot and Stackelberg periods. This is because in the former type of period wages respond less than fully to the current shock, and in the latter they do not respond to the current shock at all.

Note that if the conditions of the proposition hold then social welfare increases unambiguously with a more explicit IT, because the utility of both the policymaker and the public increases. In addition to improved macroeconomic outcomes that benefit both players, the public's wage bargaining cost is lower with a higher r_T.

6. Robustness and Extensions

In order to show that our findings are robust, the working paper version of this article discusses eleven alternative specifications and assumptions. Here we will only briefly look at three of them.

Specification of Expectations. It should be noted that the specification π_t^e can encompass various forms of expectations. Recall that the private agents are forward looking and rationally choose the frequency of adjusting expectations. The latter is why we do not formally adopt the standard specification $E_t\pi_{t+1}$ in the Phillips curve - but the outcomes are analogous. This can be seen in equation (7), in which the equilibrium outcomes are identical to Clarida et al. (1999).

The reason this timing does not play a qualitative role is that if the inflation target is sufficiently explicit, $r_T > \tilde{r}_T$, then the policymaker is no longer tempted to surprise inflate/deflate. The public then expects optimal inflation to be played throughout, ie in all periods. Since it has full information, various specifications of expectations will be equivalent in the absence of shocks, and in their presence only different quantitative, not qualitatively. Therefore, our analysis preserves the forward looking nature of the Clarida et al. (1999) model.

Anchored Expectations vs Anchored Wages. While our analysis explicitly modelled anchorness of wages, it is straightforward to show that all the results analogously obtain for anchorness of expectations as well. This is intuitive – wages and expectations enter the model (the economy as well as the public's preferences) in the same way.

To demonstrate, let us make the cost of updating expectations positive $C_e > 0$. It is again natural to assume it to be decreasing in the frequency of updating, $\frac{\triangle C_e}{\triangle r_e} < 0$, which reflects the cost of processing information (see eg the discussion in Mankiw and Reis (2002)). In line with this, we can consider any $\gamma \in (0, 1]$ and/or $\kappa > 0$. It is evident that if the updating cost is sufficiently high, $C_e \geq \tilde{C}_e$, where the threshold $\tilde{C}_e$ is an increasing function of β in (5), then $r_e^* > 1$, ie the public will find it optimal to update expectations infrequently and leave them anchored at the target level to minimize the cost.

This is shown formally in Libich (2009), and is consistent with the models of (i) rational inattention (Sims (2003) and Reis (2006)), (ii) 'economically rational expectations' (Feige and Pearce (1976)), as well as models that examine some sort of inertia/stickiness/rigidity in updating/forming expectations (see eg Ball (2000), Mankiw and Reis (2002), Carroll and Slačálek (2006), Morris and Shin (2006)). In these papers agents' expectation formation is an (explicit or implicit) result of cost-benefit calculations.

It is straightforward to see that for a relatively small β the public will choose to update expectations only in the wage negotiation periods, $r_e^* = r_w^*$. This is because under a small β expectations are updated primarily to set wages accurately, and hence agents would not waste resources in Stackelberg periods in which wages cannot be adjusted.

Timeless Perspective Commitment. In terms of the $r_i^* = 1$ result reported in Proposition 3, it should be noted that the same would obtain in the standard New Keynesian setting with both forward and backward looking expectations – it is optimal for the policymaker to be able to respond to shocks the period they occur. Nevertheless, this does not prescribe the exact form of the targeting rule the policymaker should follow, ie it is compatible with both discretionary and *pre-commitment* solutions ('timeless perspective' as popularized by Woodford (1999) or quasi-commitment of Schaumburg and Tambalotti (2007)). Under the pre-commitment solutions, the targeting rule in (16) would be altered to also include past output gap, but $r_i^* = 1$ would still apply. Put differently, under our explicit commitment the restriction relates to *when* the policymaker can move, whereas under the pre-commitment solutions it relates to *how* he can move.

It is however interesting to note that both commitment concepts impact macroeconomic outcomes in the same direction. They both anchor wages/expectations to change less than would otherwise be the case. Therefore both types of commitment improve the SR variability tradeoff and reduce the volatility of interest rates.

For a discussion of additional extensions and modifications, namely: (i) the players' impatience, (ii) backward-looking expectations, (iii) heterogeneous private agents (or Unions), (iv) consumption smoothing, (v) the horizon of IT, (vi) non-discrete r_m, (vii) probabilistic r_m, and (viii) non-constant (time-varying) r_m, see the working paper version.

7. Summary and Conclusions

The paper provides a new, game theoretic tool to examine the impact of a commitment to an explicit long-run inflation target on the policymaker's short-run flexibility, and the resulting stabilization outcomes. In contrast to the widespread belief - one that Woodford coined 'traditional prejudice of central bankers' - we show formally that a legislated numerical target may increase rather than decrease stabilization flexibility. As such, it can improve the variability tradeoff through its anchoring effect on the public's behaviour.

This is because a sufficiently explicit long-term inflation target improves policy credibility under a range of circumstances. The fact that it is more visible and accountable means that it is less likely to be reneged upon. The public therefore adjusts wages and expectations less frequently in order to minimize the cost associated with their revision without fear of being surprised by inflation. Put differently, the public finds it optimal to look through shocks, and such anchored wages and expectations (at the target level) then enable the policymaker to more effectively fine-tune the economy, and do so with smaller changes in the interest rate instrument. This reduces the volatility of both inflation and output.[24]

Our analysis thus formalizes the arguments on the effects of explicit inflation targets, eg Bernanke (2003), Goodfriend (2003), McCallum (2003), Mishkin (2004), or Lacker (2005), and offers an alternative channel to that of Orphanides and Williams (2005). Similarly to these papers, it implies that the Federal Reserve, the European Central Bank, and the Bank of Japan should more explicitly commit to a long-run inflation target.

We show, in the working paper version of this article, that while the empirical evidence is not conclusive, there exists fair support for all our findings. Most importantly,

[24] The following quote by the central banker who 'taught' IT to the world, ex-governor of the Reserve Bank of New Zealand Donald Brash (2002), summarizes our findings in several respects – it stresses the desirability of (i) agents' looking through shocks, (ii) anchorness of wages/expectations, and the resulting (iii) improved stabilization, and (iv) lower interest rate volatility: *'To put it bluntly, if the Reserve Bank is to be able to 'look through' the impact of things like the increase in petrol and cigarette prices in implementing monetary policy, we New Zealanders also need to 'look through' the impact of those things on the CPI. To the extent that we don't, and instead seek compensation for the impact of those things on the CPI, the Bank will need to tighten monetary policy to a greater extent... In recent years, the Reserve Bank has been happy to report that inflationary expectations are now well anchored at a low level. We have been able to say that, as a result, we expect that smaller adjustments in interest rates will be required...'*

explicit inflation targets have been shown to: (i) anchor expectations (eg Gürkaynak et al. (2009), Beechey et al. (2008), Gürkaynak et al. (2005), Levin et al. (2004) or Kuttner and Posen (1999)), (ii) lead to either no change or a decrease in output volatility (eg Corbo, Landerretche and Schmidt-Hebbel (2001)), Arestis, Caporale and Cipollini (2002), Fatas, Mihov and Rose (2004)), and (iii) reduce the nominal interest rate and its volatility to a larger extent than non-IT countries (eg Siklos (2004), Neumann and von Hagen (2002)).[25]

Our paper implies two explanations for why some of the empirical evidence may be inconclusive. First, the above described beneficial effects of explicit IT are shown to occur under most - but not all - circumstances. Second, explicit inflation targets work in conjunction with the conventional channels of improved monetary policy such as reputation, solid past performance, central bank independence, and effective communication. Therefore, explicit IT is not a panacea; some conditions need to be satisfied for it to work effectively, and it does not work alone. We have derived the conditions under which it anchors expectations and wages, and improves the variability tradeoff in our New Keynesian model.

The main issue the analysis is unable to incorporate is the exact specification of the inflation objective. The build up of the global financial crisis showed that focusing solely on consumer prices, which is what real world IT countries have commonly done, may be problematic under some circumstances. More research is needed to deepen our understanding of whether various asset prices should play a role in setting monetary policy. Nevertheless, the above analysis demonstrated that regardless of the adopted specification of the target, it should be made sufficiently explicit in order to deliver anchored expectations and wages, and hence help the policymaker gain greater leverage over the real rate and improve stabilization outcomes.

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[25] Furthermore, it has been forcefully argued that even central banks with a legal unitary or hierarchical mandate (in which price stability is the sole or primary goal) attempt to stabilize output in practice, see eg Cecchetti and Ehrmann (1999) or Kuttner (2004).

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Appendix A. Proof of Proposition 1

Proof. Let us start by realizing that under $r_T \leq r_w$ there will be no change in the outcomes of the static game. The time-inconsistency feature will still obtain since the public cannot punish the policymaker for deviating from the O inflation level; in fact the incentive to deviate is even stronger as a surprise would last for more periods.

Therefore, we can focus on the case $r_T > r_w$. Solving backwards, the public's optimal trend wages between its last move in the dynamic stage game (that occurs in period $t = r_T - r_w + 1$) and its second move (that occurs in period $t = r_w + 1$) will be the best response to the policymaker's initial observable move. Denoting BR to be the best response, this implies $w^*_{t\in[r_w+1,r_T]} \in BR(\pi_1^T)$. The same is true for the public's initial move that is however made under imperfect information, and therefore expectations about it need to be formed. From (5) and $r_e = 1$ we know that $\pi_t^e = \pi_t, \forall t$, which the public uses to perfectly predict the policymaker's initial move and set wages accordingly, $w^*_{t\in[1,r_w)} \in BR(\pi_1^{T*})$. We therefore need to determine the policymaker's optimal play in period 1, π_1^{T*}, which will then obtain for the rest of the dynamic stage game.

The starting point is to note that in period 1 optimal wages are always set equal to inflation expectations, $w_1^* = \pi_1^e, \forall r_w$. For the optimal inflation target to be *time consistent* (ie for a Ramsey SPNE to exist), it is required that π^O be the best response to optimal wages, $\pi_1^O \in BR(w_1^O)$. This is guaranteed by the following necessary credibility condition

$$\underbrace{ar_T}_{(\pi^O,w^O)} \geq \underbrace{cr_w}_{(\pi^S,w^O)} + \underbrace{d(r_T - r_w)}_{(\pi^S,w^S)}. \tag{17}$$

Both the left-hand side (LHS) and the right-hand side (RHS) are derived under the public playing w_1^O. The LHS expresses the fact that if the policymaker plays π_1^O then he will achieve the payoff a in all $M = r_T$ periods. In contrast, the RHS describes the scenario of the policymaker playing π_1^S and initially gaining the desired output x^T through an inflation surprise, and the c payoff. This however only lasts for r_w periods. Then in period $t = r_w + 1$ the public switches to w^S which punishes the policymaker with a d payoff for the rest of the dynamic stage game, $(r_T - r_w)$ periods. Substituting in the respective values $\{a, c, d\}$ from the payoff matrix in (12) yields the necessary credibility condition (13).

To achieve credibility of the O level inflation target with certainty it is however required that *any* SPNE be Ramsey otherwise S level wages and/or expectations could occur as π^S may also be played in equilibrium. For this to be the case π_1^O must be a strictly dominant strategy, thus in addition to $\pi_1^O \in BR(w_1^O)$ from (17) it is required that π_1^O is the *unique* best response to w^S, ie $BR(w_1^S) = \{\pi_1^O\}$. The following condition, derived in the same way as (17) but under the public playing w_1^S, ensures this

$$(18) \qquad \underbrace{br_w}_{(\pi^O, w^S)} + \underbrace{a(r_T - r_w)}_{(\pi^O, w^O)} > \underbrace{dr_T}_{(\pi^S, w^S)} .$$

If satisfied, the policymaker prefers to play π_1^O even if he knows with certainty that w_1^S will be played and hence he will suffer some temporary output cost b due to lack of credibility. He does so knowing that he will be 'rewarded' by the public's switching to w^O when it first gets a chance, ie after r_w periods. Rearranging (18) yields the sufficient credibility condition (14) which, combined with the fact that $\tilde{r}_T$ is a function of α with the desired positive sign, completes the proof. □

Appendix B. Proof of Proposition 2

Proof. The proof of Proposition 1 derived the players' optimal choices and equilibrium outcomes for given values of r_T and r_w. Using these and moving backwards, let us examine the public's optimal r_w choice made in period 1, observing r_T. This choice entails a tradeoff between the inflation and the wage bargaining cost. If the public selects sufficiently short wage contracts, $r_w \leq \tilde{r}_w$, where $\tilde{r}_w = \frac{\lambda^2 r_T}{3\alpha}$ follows from (14), then it will uniquely ensure the Ramsey type SPNE. While its inflation cost will then be zero, $C_\pi = 0$ (due to π^{O*}), its wage bargaining cost will be higher due to more frequent bargaining - recall that $C_w = \frac{c_w}{r_w}$. Alternatively, if the public selects $r_w > \tilde{r}_w$ then the sub-optimal inflation level obtains, π^{S*}, and the public will suffer a positive inflation cost, $c_\pi > 0$, accompanied by a lower wage bargaining cost C_w. Whichever the public chooses depends on the relative magnitudes of the c_w and c_π costs.

Due to the monotonocity of C_w it is apparent that to minimize the wage bargaining cost the public would choose the upper thresholds in each of these two intervals, ie either $r_w = \tilde{r}_w = \frac{\lambda^2 r_T}{3\alpha}$ or $r_w \to \infty$. Using the public's utility function (5) with the results of Proposition 1 implies that $U_t^p(r_w = \tilde{r}_w) > U_t^p(r_w \to \infty)$ holds, $\forall r_w, r_T, t$, iff

$$c_\pi > \tilde{c}_\pi = \frac{3\alpha c_w}{\lambda^2 r_T}.$$

If satisfied, the public will choose the former scenario with the level stated in (15), $r_w^* = \tilde{r}_w = \frac{\lambda^2 r_T}{3\alpha}$. The fact that r_w^* is an increasing function of r_T completes the proof.[26] □

[26] If $\frac{\lambda^2}{3\alpha} \notin \mathbb{N}$ the public will select the highest integer below r_w^* due to the assumed $n \in \mathbb{N}$.

Appendix C. Proof of Proposition 3

Proof. We need to show that under $r_i > 1$ there exist circumstances that lead to (π^*, x^*) deviating from (16). Take for example some $r_w > 1$, focus on the first two periods of the game, and consider a sole supply shock in period 1 and no shock in period 2, ie $\hat{u}_1 \neq \hat{u}_2 = \hat{g}_1 = \hat{g}_2 = 0$. The optimal outcomes according to (16) are π_1^*, x_1^*, $\pi_2^* = \rho\pi_1^*$, and $x_2^* = \rho x_1^*$. However, under $r_i > 1$ the policymaker cannot adjust the interest rate in period 2, $i_2 = i_1$. Therefore, if the policymaker chooses outcomes according to (16) in period 1, π_1^* and x_1^*, then in period 2 the IS curve yields, using $w_2 = w_1$ (implied by $r_w > 1$), $x_2 = x_1^* \neq x_2^*$. In words, the second period output is not optimal, which completes the proof. All these statements can be seen formally in (20). □

Appendix D. Proof of Proposition 4

Proof. Let us first realize that in the SR game it is still true that $\pi_t^e = \pi_t, \forall t$, which follows from $r_e = 1$ and complete information. Despite this the wage can now be, under $r_w > 1$, at a disequilibrium level, $w_t \neq \pi_t$, due to shocks. We will denote the Stackelberg periods in which wage inflation cannot be adjusted by upper 'frown', eg $\overset{\frown}{\pi}_t$ (the remaining Cournot periods will have no extra notation).

The effect of r_w on stabilization outcomes will be examined in four steps. We will first show that the variability of inflation and output in Cournot periods is decreasing in r_w. Second, we show the same for Stackelberg periods. Third, as the Stackelberg variance may be higher than Cournot, the same will be demonstrated for average variance with the weight being the relative occurrence of Cournot vs Stackelberg periods. This will prove the result for all $r_w \geq 2$. Fourth, this average variance will be shown to be less than variance under $r_w = 1$, which will extend the proof to all $r_w \geq 1$.

Step 1. It follows from (5) that in Cournot periods optimal wages are set as an average of expected inflation over the whole duration of the contract

$$w_t^* = \frac{1}{r_w} \sum_{s=0}^{r_w - 1} E_t \pi_{t+s} = \frac{\pi_t}{r_w} \sum_{s=0}^{r_w - 1} \rho^s, \tag{19}$$

where the second element uses the fact that the public rationally expects $E_t \pi_{t+1} = \rho\pi_t$. Substituting (16)-(19) into the Phillips curve implies the reduced form expressions for equilibrium inflation and the output gap in Cournot periods

$$\pi_t^* = \frac{1}{\left(\frac{\lambda^2}{\alpha} + 1\right) - \frac{1}{r_w} \sum\limits_{s=0}^{r_w - 1} \rho^s} u_t \quad \text{and} \quad x_t^* = -\frac{\lambda}{\alpha} \pi_t^*. \tag{20}$$

It is straightforward to see in (20) that the stabilization tradeoff in Cournot periods improves in wage anchorness. Focusing on the last element of the denominator, it is required that

$$\frac{\triangle \left(\frac{1}{r_w} \sum\limits_{s=0}^{r_w - 1} \rho^s \right)}{\triangle r_w} < 0. \tag{21}$$

This, using the formula for a finite sum and rearranging, yields

$$\rho^{r_w}[r_w(1-\rho)+1] < 1, \tag{22}$$

which holds for all assumed values of r_w and ρ. Denoting the variances of Cournot inflation and the output gap (conditional only on the fact that they occur in Cournot periods) by σ_π^2 and σ_x^2 we therefore have

$$\frac{\triangle\sigma_\pi^2}{\triangle r_w} < 0 \quad \text{and} \quad \frac{\triangle\sigma_x^2}{\triangle r_w} = \left(\frac{\lambda}{\alpha}\right)^2 \frac{\triangle\sigma_\pi^2}{\triangle r_w} < 0,$$

ie the variability of both inflation and output in Cournot periods is decreasing in r_w.

Step 2. In Stackelberg periods wage inflation is still at the level set at the preceding Cournot period, $\widehat{w}_t = w_{t-1-s}$, where $s = 0, 1, ..., r_w - 1$. Using this together with (16)-(19) and the Phillips curve yields the expressions for Stackelberg inflation and the output gap

$$\widehat{\pi}_{t+1+s} = \frac{\alpha}{\alpha+\lambda^2}\left(\frac{1}{r_w}\pi_t \sum_{s=0}^{r_w-1}\rho^s + u_{t+1+s}\right) \quad \text{and} \quad \widehat{x}_{t+1+s} = -\frac{\lambda}{\alpha}\widehat{\pi}_{t+1+s},$$

which implies the following variance of Stackelberg inflation and the output gap conditional on the type of period and the time after the Cournot period s (denoted by $\widehat{\sigma}_{\pi_s}^2$ and $\widehat{\sigma}_{x_s}^2$)

$$\begin{aligned} \widehat{\sigma}_{\pi_s}^2 &= \left[\left(\tfrac{\alpha}{(\alpha+\lambda^2)r_w}\sum_{s=0}^{r_w-1}\rho^s\right)^2 + \left(\tfrac{\alpha}{\alpha+\lambda^2}\right)^2 + \sum_{s=0}^{r_w-1}\rho^s \tfrac{2\alpha^2}{(\alpha+\lambda^2)(r_w+1)}\rho^{s+1}\right]\sigma_u^2, \\ \widehat{\sigma}_{x_s}^2 &= \left(\tfrac{\lambda}{\alpha}\right)^2\left[\left(\tfrac{\alpha}{(\alpha+\lambda^2)r_w}\sum_{s=0}^{r_w-1}\rho^s\right)^2 + \left(\tfrac{\alpha}{\alpha+\lambda^2}\right)^2 + \sum_{s=0}^{r_w-1}\rho^s \tfrac{2\alpha^2}{(\alpha+\lambda^2)(r_w+1)}\rho^{s+1}\right]\sigma_u^2. \end{aligned} \tag{23}$$

Using (21)-(22) from Step 1, equation (23) implies that the variability in any one Stackelberg period is also decreasing in r_w, namely $\frac{\triangle\widehat{\sigma}_{x_s,\pi_s}^2}{\triangle r_w} < 0$.

Step 3. Note that the variability of inflation and output in both types of periods only differs by a constant factor of $(\frac{\lambda}{\alpha})^2$. Therefore, we will show all the results for the output gap only, which will prove the claims for inflation as well. Let us now show the same to hold for the *average* per period output variance in Stackelberg periods, denoted by $\bar{\widehat{\sigma}}_x^2$, which is the following weighted average

$$\bar{\widehat{\sigma}}_x^2 = \frac{1}{r_w-1}\sum_{s=1}^{r_w-1}\widehat{\sigma}_{x_s}^2. \tag{24}$$

The fact that the covariance between various periods' shocks is increasing in ρ implies that if $\frac{\triangle\bar{\widehat{\sigma}}_x^2}{\triangle r_w} < 0$ holds for ρ arbitrarily close to 1, then it holds for all $\rho = [0, 1)$. Using therefore (23) and (24) with $\rho = 1$ and taking the first difference with respect to r_w yields

$$\frac{\triangle\bar{\widehat{\sigma}}_x^2}{\triangle r_w} = -2\frac{\lambda^2}{(\alpha+\lambda^2)^2(r_w-1)^2} < 0.$$

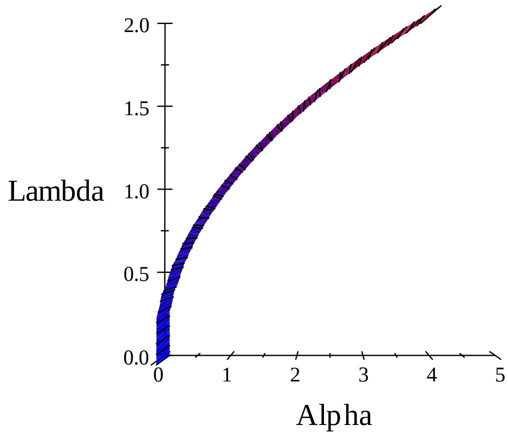

FIGURE 3. Plot of the solution to (25) in the (α, λ) parameter space. All points to the right of the curve satisfy the condition.

Combining this with $\frac{\triangle\sigma_x^2}{\triangle r_w} < 0$ implies that the average *overall* output variance under $r_w \geq 2$, denoted $\bar{\sigma}_x^2$, is decreasing in r_w.

Step 4. This step shows that the conclusion of Step 3 extends to the case of $r_w = 1$, ie our result in Proposition 4 applies for $\forall r_w$. We need to show that $\bar{\sigma}_x^2(r_w) < \sigma_x^2(r_w = 1), \forall r_w$. To do so we will focus on deriving a sufficient condition, and hence use the worst case scenario with $\rho \to 1$ and $r_w = 2$ (the result $\frac{\Delta\bar{\sigma}_x^2}{\Delta r_w} < 0$ implies that $\bar{\sigma}_x^2$ has its maximum in $r_w = 2$ in the region for which it is defined, $r_w \geq 2$). Using these values with all the above information yields the following

$$\sigma_x^2(r_w = 1) - \bar{\sigma}_x^2(r_w = 2) = \frac{1}{\lambda^2} - \frac{1}{2}\lambda^2 \left(\frac{6}{(\alpha + \lambda^2)^2} + \frac{1}{(\frac{\alpha}{2} + \lambda^2)^2} \right) > 0. \tag{25}$$

This inequality can be simplified into $4\alpha^4 + 24\alpha^3\lambda^3 + 39\alpha^2\lambda^4 + 12\alpha\lambda^6 - 12\lambda^8 > 0$, which makes it transparent that there exists some threshold value $\tilde{\alpha}$ above which it is satisfied (see Figure 3 for a graphical depiction).

Since the real world value of λ is very small, it can be claimed that (25), and hence Proposition 4 (given that the expression for inflation is analogous), holds for all 'reasonable' parameter values.[27] Steps 2-4 also imply that the policy frontier of Figure 2 is qualitatively unchanged if we replace the Cournot variances, σ_π and σ_x, with the average variances, $\bar{\sigma}_\pi$ and $\bar{\sigma}_x$. This completes the proof. □

[27] For example, if $\lambda = 0.13$ as estimated by Rudebusch (2002), then the sufficient condition in (25) is satisfied for $\alpha \geq \tilde{\alpha} = 0.017$. The necessary and sufficient condition is yet weaker; under $\lambda = 0.13, \rho = 0.2, r_w = 12$ we have $\tilde{\alpha} \cong 0.0008$.

Appendix E. Proof of Proposition 5

Proof. The public's average one-period expected utility is, using (5), the negative of the sum of the wage bargaining cost and the average variance of the inflation-wage gap, denoted $\bar{\sigma}_p^2$

$$E\bar{U}^p = -\frac{c_w}{r_w} - \bar{\sigma}_p^2.$$

Substituting in the above derived expressions and rearranging yields

$$E\bar{U}^p = -\frac{c_w+1}{r_w}\left(\frac{\alpha\left(1-\frac{\sum_{s=0}^{r_w-1}\rho^s}{r_w}\right)^2\sigma_u^2}{\left(\alpha+\lambda^2-\frac{\alpha}{r_w}\right)^2} + \sum_{s=1}^{r_w-1}\left(\begin{array}{c}\frac{\alpha^2\sigma_u^2}{\left(\alpha+\lambda^2-\frac{\alpha}{r_w}\right)^2}\left(\frac{-\lambda^2\sum_{s=0}^{r_w-1}\rho^s}{(\alpha+\lambda^2)r_w}\right)^2+\left(\frac{\alpha\sigma_u}{\alpha+\lambda^2}\right)^2\\ +\frac{2\alpha^2\rho^s\sigma_u^2}{(\alpha+\lambda^2)(\alpha+\lambda^2-\frac{\alpha}{r_w})}\left(\frac{-\lambda^2\sum_{s=0}^{r_w-1}\rho^s}{(\alpha+\lambda^2)r_w}\right)\end{array}\right)\right).$$

We aim to derive a *sufficient* condition for the level of $\tilde{c}_w$, above which the public's expected utility is *monotonically* increasing in $r_w, \forall r_w \geq 1$. It therefore suffices to focus on the worst case scenario. Realizing that $\frac{\triangle\bar{\sigma}_p^2}{\triangle r_w} < 0$ and $\frac{\partial\bar{\sigma}_p^2}{\partial\rho} < 0$ means depicting the case $r_w = 2$ and $\rho = 0$. This yields, after some manipulations

$$c_w \geq \tilde{c}_w = \frac{2\alpha^2(\alpha^2+3\alpha\lambda^2+3\lambda^4)}{(\alpha^2+3\alpha\lambda^2+2\lambda^4)^2}\sigma_u^2. \tag{26}$$

It is straightforward to verify that the RHS of (26) has a global maximum at the level of $\lambda = 0$. Using (26) with $\lambda = 0$ one obtains the following sufficient condition

$$c_w \geq \tilde{c}_w = 2\sigma_u^2.$$

These calculations however imply that this sufficient condition is not tight, eg under $\lambda = 0.13, \rho = 0.2, r_w = 12$ and any $\alpha \leq 2$, a tighter sufficient (but still not necessary) condition becomes $c_w \geq \tilde{c}_w = 1.07\sigma_u^2$. □

Appendix F. Proof of Proposition 7

Proof. The claimed effect of r_T on $var(x^*)$ and $var(\pi^*)$ is implied by combining the fact that sufficiently explicit IT anchors wages (Proposition 5), and wage anchorness reduces the variability of both inflation and output (Proposition 4). In terms of the effect of r_T on $var(i^*)$, recall that the variability of equilibrium wages, inflation and output is decreasing in r_w. This implies, using the IS curve, that the same is true for the equilibrium interest rate, $\frac{\triangle var(i^*)}{\triangle r_w} < 0$. It then follows that $\frac{\triangle var(i^*)}{\triangle r_T} < 0$. □